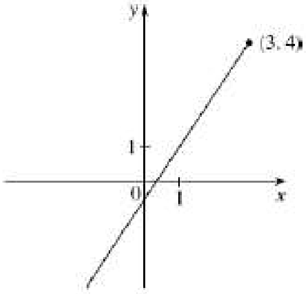
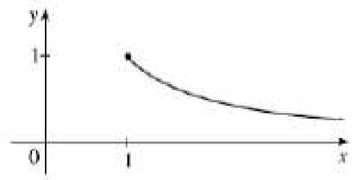
Graphing, Part 2

p. 304: 21-33 odd, 53-67 odd, 69-72, 79-80

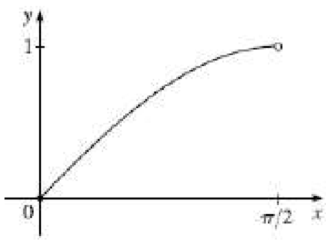
21. $f(x) = \frac{1}{2}(3x - 1),\ x \le 3$. Absolute maximum $f(3) = 4$; no local maximum. No absolute or local minimum.



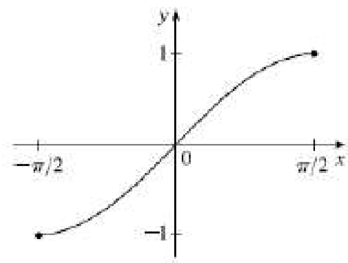
23. $f(x) = 1/x,\ x \ge 1$. Absolute maximum $f(1) = 1$; no local maximum. No absolute or local minimum.



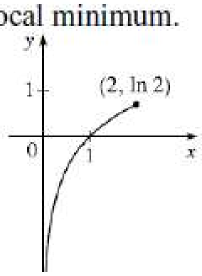
25. $f(x) = \sin x,\ 0 \le x < \frac{\pi}{2}$. No absolute or local maximum. Absolute minimum $f(0) = 0$; no local minimum.



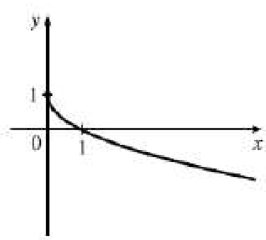
27. $f(x) = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Absolute maximum $f(\frac{\pi}{2}) = 1$; no local maximum. Absolute minimum $f(-\frac{\pi}{2}) = -1$; no local minimum.



29. $f(x) = \ln x,\ 0 < x \le 2$. Absolute maximum $f(2) = \ln 2 \approx 0.691$; no local maximum. No absolute or local minimum.

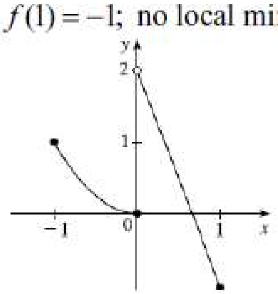


31. $f(x) = 1 - \sqrt{x}$. Absolute maximum $f(0) = 1$; no local maximum No absolute or local minimum.



33. $f(x) = \begin{cases} x^2 & \text{if } -1 \le x \le 0 \\ 2 - 3x & \text{if } 0 < x \le 1 \end{cases}$

No absolute or local maximum. Absolute minimum $f(1) = -1$; no local minimum.



53. $f(x) = 12 + 4x - x^2,\ [0,5]$. $f'(x) = 4 - 2x = 0 \Rightarrow x = 2$. $f(0) = 12, f(2) = 16$, and $f(5) = 7$. Therefore, $f(2) = 16$ is the absolute maximum value and $f(5) = 7$ is the absolute minimum value.

55. $f(x)=2x^3-3x^2-12x+1,\ [-2,3]$. $f'(x)=6x^2-6x-12=6(x^2-x-2)=6(x-2)(x+1)=0\Rightarrow$ $x=2,-1$. $f(-2)=-3, f(-1)=8, f(2)=-19$ and $f(3)=-8$. Therefore, $f(-1)=8$ is the absolute maximum value and $f(2)=-19$ is the absolute minimum value.

57. $f(x)=3x^4-4x^3-12x^2+1,\ [-2,3]$. $f'(x)=12x^3-12x^2-24x=12x(x^2-x-2)$ $=12x(x+1)(x-2)=0\Rightarrow x=-1,0,$ or 2. $f(-2)=33, f(-1)=-4, f(0)=1, f(2)=-31$ and $f(3)=28$. So, $f(-2)=33$ is the absolute maximum value and $f(2)=-31$ is the absolute minimum value.

59. $f(x)=x+\dfrac{1}{x},\ [0.2,\ 4]$. $f'(x)=1-\dfrac{1}{x^2}=\dfrac{x^2-1}{x^2}=\dfrac{(x-1)(x+1)}{x^2}=0\Rightarrow x=\pm1$, but $x=-1$ is not in the given interval. $f'(x)$ does not exist when $x=0$, but 0 is not in the given interval, so 1 is the only critical number. $f(0.2)=5.2, f(1)=2,$ and $f(4)=4.25$. So, $f(0.2)=5.2$ is the absolute maximum value and $f(1)=2$ is the absolute minimum value.

61. $f(t)=t-\sqrt[3]{t},\ [-1,4]$. $f'(t)=1-\frac{1}{3}t^{-2/3}=1-\dfrac{1}{3t^{2/3}}$. $f'(t)=0\Rightarrow 1=\dfrac{1}{3t^{2/3}}\Leftrightarrow t^{2/3}=\dfrac{1}{3}\Leftrightarrow$

$t=\pm\left(\dfrac{1}{3}\right)^{3/2}=\pm\sqrt{\dfrac{1}{27}}=\pm\dfrac{1}{3\sqrt{3}}=\pm\dfrac{\sqrt{3}}{9}$. $f'(t)$ does not exist when $t=0$. $f(-1)=0,\ f(0)=0,$

$f\left(\dfrac{-1}{3\sqrt{3}}\right)=\dfrac{-1}{3\sqrt{3}}-\dfrac{-1}{\sqrt{3}}=\dfrac{-1+3}{3\sqrt{3}}=\dfrac{2\sqrt{3}}{9}\approx 0.3849,\ f\left(\dfrac{1}{3\sqrt{3}}\right)=\dfrac{1}{3\sqrt{3}}-\dfrac{1}{\sqrt{3}}=-\dfrac{2\sqrt{3}}{9}$, and

$f(4)=4-\sqrt[3]{4}\approx 2.4125$. So $f(4)=4-\sqrt[3]{4}$ is the absolute maximum value and $f\left(\dfrac{1}{3\sqrt{3}}\right)=-\dfrac{2\sqrt{3}}{9}$ is the absolute minimum value.

63. $f(t)=2\cos t+\sin 2t, [0,\pi/2]$.

$f'(t)=-2\sin t+\cos 2t\cdot 2=-2\sin t+2(1-2\sin^2 t)=-2(2\sin^2 t+\sin t-1)=-2(2\sin t-1)(\sin t+1)$.

$f'(t)=0\Rightarrow \sin t=\frac{1}{2}$ or $\sin t=-1\Rightarrow t=\frac{\pi}{6}$. $f(0)=2,\ f\left(\frac{\pi}{6}\right)=\sqrt{3}+\frac{1}{2}\sqrt{3}=\frac{3}{2}\sqrt{3}\approx 2.598$, and $f\left(\frac{\pi}{2}\right)=0$. So $f\left(\frac{\pi}{6}\right)=\frac{3}{2}\sqrt{3}$ is the absolute maximum value and $f\left(\frac{\pi}{2}\right)=0$ is the absolute minimum value.

65. $f(x)=x^{-2}\ln x,\ \left[\frac{1}{2},4\right]$. $f'(x)=x^{-2}\cdot\dfrac{1}{x}+(\ln x)\left(-2x^{-3}\right)=x^{-3}-2x^{-3}\cdot\ln x=x^{-3}(1-2\ln x)=\dfrac{1-2\ln x}{x^3}$.

$f'(t)=0\Leftrightarrow 1-2\ln x-0\Leftrightarrow 2\ln x=1\Leftrightarrow \ln x=\frac{1}{2}\Leftrightarrow x=e^{1/2}\approx 1.649$. $f'(x)$ does not exit when $x=0$, which is not in the given interval $\left[\frac{1}{2},4\right]$. $f\left(\frac{1}{2}\right)=\dfrac{\ln\frac{1}{2}.}{\left(\frac{1}{2}\right)^2}=\dfrac{\ln 1-\ln 2}{\frac{1}{4}}=-4\ln 2\approx -2.773,$

$f\left(e^{1/2}\right)=\dfrac{\ln e^{1/2}.}{\left(e^{1/2}\right)^2}=\dfrac{\frac{1}{2}}{e}=\dfrac{1}{2e}\approx 0.184,$ and $f(4)=\dfrac{\ln 4.}{4^2}=\dfrac{\ln 4}{16}\approx 0.0866$. So $f\left(e^{1/2}\right)=\dfrac{1}{2e}$ is the absolute maximum value and $f\left(\frac{1}{2}\right)=-4\ln 2$ is the absolute minimum value.

67. $f(x)=\ln(x^2+x+1),\ [-1,1]$. $f'(x)=\dfrac{2x+1}{x^2+x+1}=0\Leftrightarrow x=-\frac{1}{2}$. Since $x^2+x+1>0$ for all x, the domain of f and f' is $\mathbb{R}$. $f(-1)=\ln 1=0,\ f\left(-\frac{1}{2}\right)=\ln\frac{3}{4}\approx -0.288$, and $f(1)=\ln 3\approx 1.099$. So $f(1)=\ln 3$ is the absolute maximum value and $f\left(-\frac{1}{2}\right)=\ln\frac{3}{4}$ is the absolute minimum value.

69. $f(x)=\frac{x^2}{e^x}\Rightarrow f'(x)=\frac{e^x(2x)-x^2(e^x)}{e^{2x}}=\frac{e^x(2x-x^2)}{e^{2x}}=\frac{x(2-x)}{e^x}$.

$f'(x)=0\Leftrightarrow x(2-x)=0\Leftrightarrow x=0,2.$ $f(-1)=e\approx 2.718, f(0)=0, f(2)=\frac{4}{e^2}\approx 0.541,$ and

$f(3)=\frac{9}{e^3}\approx 0.448.$ So the maximum value on the interval $[-1, 3]$ is **(B)** e.

70. $g(x)=3-2x-x^2\Rightarrow g'(x)=-2-2x=-2(x+1)=0\Rightarrow x=-1$
g' is positive for $x<-1$ and negative for $x>-1$, so $x=-1$ is relative max.
$g(-4)=-5$, $g(-1)=4$, $g(2)=-5$, but there is no absolute min. because we are using an open interval. Statements I and II are true, so **(A)**.

71. $s(t)=t^3-2t^2-4t+8\Rightarrow v(t)=s'(t)=3t^2-4t-4\Rightarrow a(t)=v'(t)=6t-4=2(3t-2)$

$a(t)=0\Rightarrow t=\frac{2}{3}.$ $v\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^2-4\left(\frac{2}{3}\right)-4=-\frac{16}{3}\approx -5.333.$

The absolute minimum velocity is **(C)** $-16/3$ ft/s.

72. $y=\frac{x-1}{x^2+1}$, on $[-4,4]\Rightarrow y'=\frac{(x^2+1)(1)-(x-1)(2x)}{(x^2+1)^2}=\frac{x^2+1-2x^2+2x}{(x^2+1)^2}=\frac{-x^2+2x+1}{(x^2+1)^2}$

y' is defined for all real x, and $y'=0\Rightarrow -x^2+2x+1=0\Rightarrow x=1\pm\sqrt{2}.$

$f(-4)=-\frac{5}{17}\approx -0.294, f\left(1-\sqrt{2}\right)=\frac{\sqrt{2}}{(1+\sqrt{2})^2+1}\approx -1.207, f\left(1+\sqrt{2}\right)=\frac{\sqrt{2}}{(1+\sqrt{2})^2+1}\approx 0.207,$ and

$f(4)=\frac{3}{17}\approx 0.176.$ Therefore, the function **(D)** $y=\frac{x-1}{x^2+1}$, on $[-4,4]$ has both a relative maximum,

$f\left(1+\sqrt{2}\right)=\frac{\sqrt{2}}{(1+\sqrt{2})^2+1}$ and a relative minimum, $f\left(1-\sqrt{2}\right)=\frac{\sqrt{2}}{(1+\sqrt{2})^2+1}$.

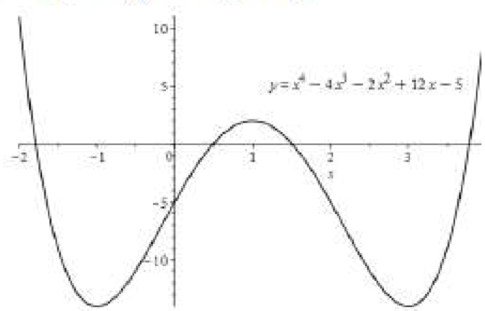
79. (a) $f(x)=x^4-4x^3-2x^2+12x-5\Rightarrow f'(x)=4x^3-12x^2-4x+12=4(x-1)(x-3)(x+1).$

$f'(x)=0\Rightarrow x=1,3,-1.$ Therefore the critical numbers of f are $x=-1,1,$ and 3.

(b) From the graph we can see that f has local minima at $x=-1$ and $x=3$. The local minima are $f(-1)=f(3)=-14$.

(c) From the graph, we can see that the local maximum value of f occurs at $x=1$, and it is $f(1)=2$.

(d) The absolute minimum is the same as the local minima, $f(-1)=f(3)=-14$.



80. (a) $R(t)=e^t-2t^2+1\Rightarrow R(2)=e^2-2(4)+1=e^2-7\approx 0.389\times 1000=389$ ft^3/min.

(b) $R'(t)=e^t-4t\Rightarrow R'(t)=0\Leftrightarrow e^t=4t\Leftrightarrow e^t/t=4\Leftrightarrow t-\ln t=\ln 4\Rightarrow t=A\approx 0.389056,$ and $t=B\approx 2.15329.$ There are two critical points, A and B, in the interval.

$R(0)=e^0+1=2$ thousand ft^3/m, $R(A)\approx 0.340$ thousand ft^3/m, $R(B)\approx 2.174$ thousand ft^3/m, and $R(3)=e^3-17\approx 3.0855$ thousand ft^3/m. Therefore, $(B,2.174)$ is a local maximum and $(A,0.340)$ is a local minimum.

(c) From the work in part (b) we see that the absolute minimum is $R(A)\approx 0.340$ thousand ft^3/m, and the absolute maximum is $R(3)=e^3-17\approx 3.0855$ thousand ft^3/m.

p. 334: 66, 70-71, 82, 89, 98

66. (a) The point $(2,3)$ is a local maximum because $f(2)=0$ and f' changes from positive to negative there. The point $(5,-2)$ is a local minimum because $f(5)$ is undefined and f' changes from negative to positive there.

(b) The point $(5,-2)$ is a point of inflection because $f''(5)$ does not exist and f'' changes from negative to positive at this point.

(c) The function is increasing on the intervals where $f'(x)>0$. Therefore, f is increasing on the intervals $[0,2]$ and $(5,\infty)$.

70. (a) $g(x)=\dfrac{\ln x}{x} \Rightarrow g'(x)=\dfrac{x\cdot\frac{1}{x}-\ln x\cdot 1}{x^2}=\dfrac{1-\ln x}{x^2}$ $g'(x)=0 \Leftrightarrow 1-\ln x=0 \Leftrightarrow 1=\ln x \Leftrightarrow e=x.$ The domain of g and g' is $x>0$. Thus, the only critical point of g is $x=e$. $g'(x)<0 \Leftrightarrow x>e$ and $g'(x)>0 \Leftrightarrow x<e$. Therefore, g has a local maximum at the point $\left(e,\frac{1}{e}\right)$.

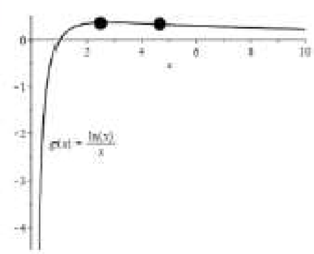
(b) $\displaystyle\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}\frac{\ln x}{x}=\lim_{x\to 0^+}\ln x\cdot\frac{1}{x}\overset{H}{=}\lim_{x\to 0^+}\frac{1}{x}\left(-\frac{1}{x^2}\right)=\lim_{x\to 0^+}\left(-\frac{1}{x^3}\right)=-\infty.$ Thus g has a vertical asymptote at $x=0$.

(c) $g'(x)=(1-\ln x)x^{-2} \Rightarrow g''(x)=(1-\ln x)\cdot(-2x^{-3})+x^{-2}\cdot(-x^{-1})=\dfrac{-2(1-\ln x)}{x^3}-\dfrac{1}{x^3}=\dfrac{2\ln x-3}{x^3}$

$g''(x)=0 \Leftrightarrow 2\ln x-3=0 \Leftrightarrow \ln x=\frac{3}{2} \Leftrightarrow x=e^{3/2}$. $g''(x)<0 \Leftrightarrow x<e^{3/2}$ and $g''(x)>0 \Leftrightarrow x>e^{3/2}$.

Thus, the only inflection point of g is $\left(e^{3/2},\dfrac{3}{2e^{3/2}}\right)$.

(d)



71. (a) $f(x)=\cos x-\cos^2 x \Rightarrow f'(x)=-\sin x-2\cos x(-\sin x)=\sin x(2\cos x-1)$

$f'(x)=0 \Leftrightarrow \sin x(2\cos x-1)=0 \Leftrightarrow \sin x=0$ or $2\cos x=1 \Leftrightarrow \cos x=\frac{1}{2}$. On the interval $\left[0,\frac{3\pi}{2}\right]$, $\sin x=0 \Leftrightarrow x=0,\pi,$ and $\cos x=\frac{1}{2} \Leftrightarrow x=\frac{\pi}{3}$ $f'(x)<0 \Leftrightarrow \frac{\pi}{3}<x<\pi$ and $f'(x)>0 \Leftrightarrow 0<x<\frac{\pi}{3}$ and $\pi<x\le\frac{3\pi}{2}$. Therefore, $x=\frac{\pi}{3}$ is a local maximum and $x=\pi$ is a local minimum.

(b) $f''(x)=\sin x(-2\sin x)+(2\cos x-1)\cos x=2\cos^2 x-\cos x-2\sin^2 x=4\cos^2 x-\cos x-2.$

If we let $u=\cos x$, we can rewrite $f''(x)=g(u)=4u^2-u-2$ and use the quadratic formula to find $u=\dfrac{1\pm\sqrt{33}}{8} \Rightarrow \cos x=\dfrac{1\pm\sqrt{33}}{8} \Rightarrow x=\arccos\left(\dfrac{1\pm\sqrt{33}}{8}\right)\approx 0.568,$ and 2.206. Using technology, we also find $x\approx 4.078$. Then $f''(x)<0 \Leftrightarrow 0.569\le x\le 2.206$ and $4.708\le x\le 3\pi/2$ and $f''(x)>0 \Leftrightarrow 0\le x\le 0.569$ and $2.206\le x\le 4.708$. Thus, f has points of inflection at $x\approx 0.569, x\approx 2.206,$ and $x\approx 4.708$.

(c) Using the endpoints and critical points we find $f(0)=0$, $f\left(\frac{\pi}{3}\right)=0.25$, $f(\pi)=-2$, and $f\left(\frac{3\pi}{2}\right)=0$. Therefore, the absolute maximum of f on $\left[0,\frac{3\pi}{2}\right]$ is $f\left(\frac{\pi}{3}\right)=0.25$, and the absolute minimum is $f(\pi)=-2$.

(d) Using the roots of f'' as critical points for f', we check the value of f' at the critical points and endpoints: $f'(0)=0$, $f'(0.569)\approx 0.369$, $f'(2.206)\approx -1.7602$, $f'(4.078)\approx 1.7602$, and $f'\left(\frac{3\pi}{2}\right)=1$. Therefore, the absolute maximum of f' on $\left[0,\frac{3\pi}{2}\right]$ is $f'(4.078)\approx 1.7602$ and the absolute minimum is $f'(2.206)\approx -1.7602$.

82. $f(x)=\sqrt{x}\Rightarrow f'(x)=\frac{1}{2}x^{-1/2}\Rightarrow f'(64)=\frac{1}{16}$ so an equation of the tangent line at the point $(64,f(64))=(64,8)$ is $y=\frac{1}{16}(x-64)+8$ or $y=\frac{1}{16}x+4$. Therefore, an approximation of $f(64.2)$ is $y=\frac{1}{16}(64.2-64)+8=8.0125$.

$f''(x)=-\frac{1}{4}x^{-3/2}=-\dfrac{1}{4\sqrt{x^3}}\Rightarrow f''(64)<0$ so f is concave down at $x=64$ and the tangent line approximation is an overestimate. Using technology, we see that $\sqrt{64.2}\approx 8.01239025$.

89. (a) The rate of increase of the population is initially very small, then gets larger until it reaches a maximum at about $t=8$ hours, and decreases toward 0 as the population begins to level off.

(b) The rate of increase has its maximum value at $t=8$ hours.

(c) The population function is concave up on $(0,8)$ and concave down on $(8,18)$.

(d) At $t=8$, the population is about 350, so the inflection points is about $(8,350)$.

98.

(a) $f(x)=6x^{2/3}-2x^{5/3}\Rightarrow f'(x)=4x^{-1/3}-\frac{10}{3}x^{2/3}\Rightarrow f''(x)=-\frac{4}{3}x^{-4/3}-\frac{20}{9}x^{-1/3}$.

(b) $f'(x)=4x^{-1/3}-\frac{10}{3}x^{2/3}=2x^{-1/3}(2-\frac{5}{3}x)=-\dfrac{2}{3x^{1/3}}(5x-6)$. $f'(0)$ does not exist, so $x=0$ is a critical value. $f'(x)=0\Leftrightarrow 5x-6=0\Leftrightarrow 5x=6\Leftrightarrow x=\frac{6}{5}$, so $x=\frac{6}{5}$ is the only other critical value. Using the Second Derivative test, $f''\left(\frac{6}{5}\right)\approx -3.137<0$ so there is a local maximum at $x=\frac{6}{5}$. We must use the First Derivative Test for $x=0$: $f'(x)$ is negative to the left of $x=0$ and positive to the right, so there is a local minimum at $x=0$.

(c) We use a sign chart for the second derivative:

Interval	$f''(x)=-\frac{4}{3}x^{-4/3}-\frac{20}{9}x^{-1/3}$	Concavity
$(-\infty,0)$	$f''(-1)\approx 0.889>0$	up
$(0,\frac{6}{5})$	$f''(1)\approx -3.556<0$	down
$(\frac{6}{5},\infty)$	$f''(2)\approx -2.293<0$	down

Therefore, f is concave up on $(-\infty,0)$.

(d) $\lim_{x\to 0^-}f'(x)=\lim_{x\to 0^-}\left[4x^{-1/3}-\frac{10}{3}x^{2/3}\right]=\lim_{x\to 0^-}\dfrac{4}{\sqrt[3]{x}}-0=-\infty$ and $\lim_{x\to 0^+}f'(x)=\lim_{x\to 0^+}\left[4x^{-1/3}-\frac{10}{3}x^{2/3}\right]$

$=\lim_{x\to 0^+}\dfrac{4}{\sqrt[3]{x}}-0=\infty$, so there is a vertical tangent at $x=0$.

p. 363: 69-73

69. $y = f(x) = x^2 \sin x \Rightarrow f'(x) = x^2 \cos x + 2x \sin x = x(x\cos x + 2\sin x) = 0 \Rightarrow x = 0$ or $x\cos x + 2\sin x = 0 \Rightarrow x + 2\dfrac{\sin x}{\cos x} = 0 \Rightarrow x + 2\tan x = 0$ Using technology, we see that $f'(x)$ has two non-zero roots, $\pm a$, in $(-\pi, \pi)$. Then we see that $f'(x) > 0 \Leftrightarrow -a < x < a$, so f is increasing on $(-a, a)$ and f is decreasing on $(-\pi, -a)$ and (a, π). Thus f has two relative extrema in this interval. Then $f''(x) = x^2(-\sin x) + 2x\cos x + 2x\cos x + 2\sin x = 2\sin x + 4x\cos x - x^2 \sin x$. Again, using technology we see $f''(x)$ that has 3 roots in $(-\pi, \pi)$, and $f''(x)$ changes sign at each of these roots. Therefore, the correct answer is (**D**), two relative extrema and three points of inflection.

70. $f(x) = \dfrac{e^x(x^2-1)}{x}$ is not defined at $x = 0$ so choices (A), (C) and (D) cannot be correct.

$$f(x) = \frac{e^x(x^2-1)}{x} = e^x\left(x - \frac{1}{x}\right) \Rightarrow f'(x) = e^x\left(1 - \frac{-1}{x^2}\right) + \left(x - \frac{1}{x}\right)e^x = e^x\left(\frac{x^2+1}{x^2} + \frac{x^2-1}{x}\right)$$

$= e^x\left(\dfrac{x^2+1+x^3-x}{x^2}\right) = e^x\left(\dfrac{x^3+x^2-x+1}{x^2}\right)$. $f'(x)$ is positive on (**B**), $[-1,0)\cup[1,\infty)$. *Note*: $f'(x)$ is also positive on $[-1,0)\cup(0,\infty)$.

71. $f(x) = x^4 + 3x^2 - 2x - 3 \Rightarrow f'(x) = 4x^3 + 6x - 2 \Rightarrow f''(x) = 12x^2 + 6 > 0$ so f is concave on its domain, $\mathbb{R}$. This is option (**A**).

72. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{2^x}{2^x - 1} = \lim\limits_{x\to\infty} \dfrac{1}{1-2^{-x}} = \dfrac{1}{1-0} = 1 \Rightarrow f$ has a horizontal asymptote at $y = 1$. (II)

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{2^x}{2^x-1} = \dfrac{0}{0-1} = 0 \Rightarrow f$ has a horizontal asymptote at $y = 0$. (III)

Finally, $\lim\limits_{x\to0^+} \dfrac{2^x}{2^x-1} = \infty$, and $\lim\limits_{x\to0^-} \dfrac{2^x}{2^x-1} = -\infty$, so f has a vertical asymptote at $x = 0$. (I) The correct choice is (**D**), I, II and III.

73. (a) $f(x) = x^4 - 2x^3 = x^3(x-2) \Rightarrow f(x) > 0$ when $x > 2$ or $x < 0$.

(b) $f'(x) = 4x^3 - 6x^2 = 2x^2(2x-3) > 0 \Leftrightarrow (2x-3) > 0 \Leftrightarrow 2x > 3 \Leftrightarrow x > \frac{3}{2}$. Therefore, f is increasing on $(\frac{3}{2}, \infty)$.

(c) $f'(x) = 4x^3 - 6x^2 \Rightarrow f''(x) = 12x^2 - 12x = 12x(x-1)$. $f''(x) = 0 \Leftrightarrow 12x(x-1) = 0 \Leftrightarrow x = 0$ or $x = 1$. $f''(x) < 0 \Leftrightarrow 0 < x < 1$, and $f''(x) > 0 \Leftrightarrow -\infty < x < 0$ and $1 < x < \infty$. So f has inflection points at $x = 0$ and $x = 1$.

(d) By the work in (c), f is concave up on $(-\infty, 0)$ and $(1, \infty)$.