Interpreting Derivatives

p. 176: 73-78

73. (a) $f'(x)$ is the rate of change of the production cost with respect to the number of ounces of gold produced. Its units are dollars per ounce.
(b) After 800 ounces of gold have been produced, the rate at which the production cost is increasing is \$17 / ounce. So the cost of producing the 800^{th} or (801^{st}) ounce is about \$17.
(c) In the short term, the values of $f'(x)$ will decrease because more efficient use is made of start-up costs as x increases. But eventually $f'(x)$ might increase due to large-scale operations.

74. (a) $f'(5)$ is the rate of growth of the bacteria population with $t = 5$ hours. Its units are bacteria per hour.
(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

75. (a) $H'(58)$ is the rate at which the daily heating cost changes with respect to temperature when the outside temperature is 58 °F. The units are dollars / $^\circ$F.
(b) If the outside temperature increases, the building should require less heating, so we would expect $H'(58)$ to be negative.

76. (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).
(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

77. (a) $S'(T)$ is the rate at which the oxygen solubility changes with respect to the water temperature. Its units are $(\text{mg/L})/^\circ\text{C}$.
(b) For $T = 16^\circ\text{C}$, it appears that the tangent line to the curve goes through the points $(0,14)$ and $(32,7)$. So $S'(16) \approx \dfrac{7-14}{32-0} = -\dfrac{7}{32} = -0.21875\ (\text{mg/L})/^\circ\text{C}$. This means that as the temperature increases past the oxygen solubility is decreasing at a rate of $0.21875\ (\text{mg/L})/^\circ\text{C}$.

78. (a) $S'(T)$ is the rate of change of the maximum sustainable speed of Coho salmon with respect to the temperature. The units of $S'(T)$ are $(\text{cm/s})/^\circ\text{C}$.
(b) For $T = 15^\circ\text{C}$, it appears the tangent line to the curve goes through the points $(10,25)$ and $(20,32)$. So $S'(15) \approx \dfrac{32-25}{20-10} = 0.7\ (\text{cm/s})/^\circ\text{C}$. This tells us that at 15°C, the maximum sustainable speed of Coho salmon is changing at a rate of $0.7\ (\text{cm/s})/^\circ\text{C}$.
In a similar fashion for $T = 25^\circ\text{C}$, we can use the point (20, 35) and (25, 25) to obtain $S'(25) \approx \dfrac{25-35}{25-20} = -2\ (\text{cm/s})/^\circ\text{C}$. As it gets warmer than 20°C, the maximum sustainable speed decreases rapidly.

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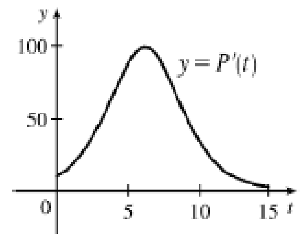
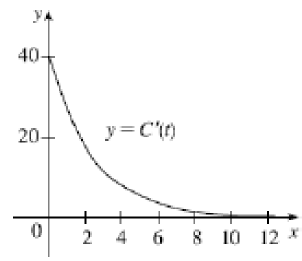
20.

The slopes of the tangent lines on the graph of $y = P(t)$ are always positive, so the y-values of $y = P'(t)$ are always positive. These values start out relatively small and keep increasing, reaching a maximum at about $t = 6$. Then the y-values of $y = P(t)$ decrease and get close to zero. The graph of P' tells us that the yeast culture grows most rapidly after 6 hours and then the growth rate declines.



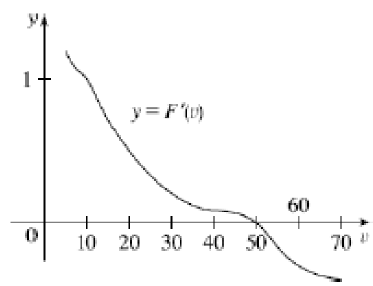
21.

(a) $C'(t)$ is the instantaneous rate of change of percentage of full capacity with respect to elapsed time in hours.
(b) The graph of $C'(t)$ tells us that the rate of change of percentage of full capacity is decreasing and approaching zero.



22.

(a) $F'(v)$ is the instantaneous rate of change of fuel economy with respect to speed.
(b) Graphs will vary depending on estimates of F' but will change from positive to negative at about $v = 50$.
(c) To save on gas, drive at the speed where F is a maximum and F' is 0, which is about 50 mi/h.



23.

$$G'(5) = \lim_{h\to 0}\frac{G(5+h)-G(5)}{h} = \lim_{h\to 0}\frac{4000-3(5+h)^2-\left(4000-3\cdot 5^2\right)}{h}$$

$$= \lim_{h\to 0}\frac{4000-75-30h-3h^2-4000+75}{h} = \lim_{h\to 0}\frac{-30h-3h^2}{h} = \lim_{h\to 0}\frac{h(-30-3h)}{h}$$

$$= \lim_{h\to 0}(-30-3h) = -30$$

(b) $G'(5)$ is the instantaneous rate of change in the amount of oil in the tank at 5:00 AM. The units $G'(t)$ of are gallons per hour. So at 5:00 AM, the oil is leaking from the tank at a rate of 30 gallons per hour.

24.

(a) $T'(8) \approx \dfrac{T(9)-T(7)}{9-7} = \dfrac{54-60}{2} = \dfrac{-6}{2} = -3$ degrees per cm.

(b) $T'(8)$ tells us that eight cm from the heated end of the rod, the temperature is decreasing at a rate of 3 degrees per cm.

47. (a) $U'(t)$ is the rate at which the unemployment rate is changing with respect to time. Its units are percent unemployed per year.

(b) To find $U'(t)$, we use $\lim_{h\to 0}\dfrac{U(t+h)-U(t)}{h} \approx \dfrac{U(t+h)-U(t)}{h}$ for small values of h.

For **2004**: $U'(2004) \approx \dfrac{U(2005)-U(2004)}{2005-2004} = \dfrac{5.1-6.0}{1} = -0.9$

For **2005**: We estimate by using $h = -1$ and $h = 1$, and then average the two results to obtain a final estimate.

$h = -1 \Rightarrow U'(2005) \approx \dfrac{U(2004)-U(2005)}{2004-2005} = \dfrac{6.0-5.1}{-1} = -0.9;$

$h = 1 \Rightarrow U'(2005) \approx \dfrac{U(2006)-U(2005)}{2006-2005} = \dfrac{4.5-5.1}{1} = -0.5.$

So we estimate that $U'(2004) \approx \frac{1}{2}\left[-0.9+(-0.5)\right] = -0.7.$

t	$U'(t)$	t	$U'(t)$	t	$U'(t)$	t	$U'(t)$
2004	–0.9	2007	0.60	2010	–0.20	2013	–0.95
2005	–0.7	2008	2.35	2011	–0.75	2014	–1.05
2006	–0.25	2009	1.90	2012	–0.75	2015	–0.90

48. (a) $N'(t)$ is the rate at which the number of minimally invasive cosmetic surgery procedures performed in the United States is changing with respect to time. Its units are thousands of surgeries per year.

(b) To find $N'(t)$, we use $\lim_{h\to 0}\dfrac{N(t+h)-N(t)}{h} \approx \dfrac{N(t+h)-N(t)}{h}$ for small values of h.

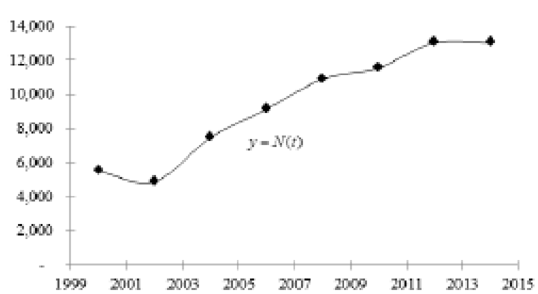
For **2000**: $N'(2000) \approx \dfrac{N(2002)-N(2000)}{2002-2000} = \dfrac{4897-5500}{2} = -301.5$

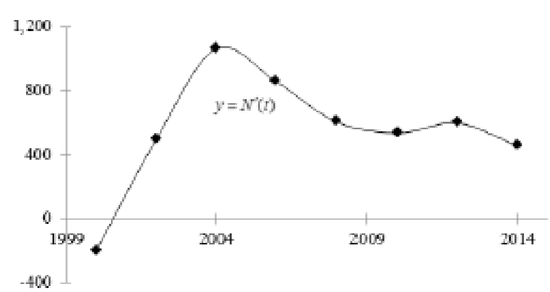
For **2002**: $N'(2002) \approx \dfrac{N(2004)-N(2000)}{2000-2000} = \dfrac{7470-5500}{4} = 492.5$

Note that this gives the same estimate that averaging two difference quotients would.

t	2000	2002	2004	2006	2008	2010	2012	2014
$N'(t)$	–301.5	492.5	1060.25	856.75	605.75	534.5	596	455

(c)





(d) We could get more accurate values for $N'(t)$ by obtaining data for more values of t.

52. dN/dp is the rate at which the number of people who travel by car to another state for vacation changes with respect to the price of gasoline. If the price of gasoline goes up, we would expect fewer people to travel, so we would expect dN/dp to be negative.