p. 103: 25-36

25. For $f(x)=\dfrac{x^2-3x}{x^2-9}$:

x	$f(x)$
3.1	0.508 197
3.05	0.504 132
3.01	0.599 832
3.001	0.500 083
3.001	0.4500 008

x	$f(x)$
2.9	0.491 525
2.95	0.495 798
2.99	0.499 165
2.999	0.499 917
2.9999	0.499 992

It appears that $\lim\limits_{x\to 3}\dfrac{x^2-3x}{x^2-9}=\dfrac{1}{2}$.

26. $f(x)=\dfrac{x^2-3x}{x^2-9}$:

x	$f(x)$
–2.5	–5
–2.9	–29
–2.95	–59
–2.99	–299
–2.999	–2999
–2.9999	–29,999

x	$f(x)$
–3.5	7
–3.1	31
–3.05	61
–3.01	301
–3.001	3001
–3.0001	30,001

It appears that $\lim\limits_{x\to -3^+}\dfrac{x^2-3x}{x^2-9}=-\infty$ and that $\lim\limits_{x\to -3^-}\dfrac{x^2-3x}{x^2-9}=\infty$ so $\lim\limits_{x\to -3}\dfrac{x^2-3x}{x^2-9}$ does not exist.

27. For $f(x)=\dfrac{x}{\sin x}$:

x	$f(x)$
0.5	1.042 915
0.1	1.001 669
0.01	1.000 017
0.001	1.000 000
0.0001	1.000 000

x	$f(x)$
–0.5	1.042 915
–0.1	1.001 669
–0.01	1.000 017
–0.001	1.000 000
–0.0001	1.000 000

It appears that $\lim\limits_{x\to 0}\dfrac{x}{\sin x}=1$.

28. For $f(x)=\ln x$:

x	$f(x)$
1.5	0.405 465
1.1	0.095 310
1.01	0.009 950
1.001	0.001 000
1.0001	0.000 100

x	$f(x)$
0.5	–0.693 147
0.9	–0.105 361
0.99	–0. 010 050
0.999	–0.001 001
0.9999	–0.000 100

It appears that $\lim\limits_{x\to 1}\ln x=0$.

29. For $f(t)=\dfrac{e^{5t}-1}{t}$:

t	$f(t)$
0.5	22.364 988
0.1	6.487 213
0.01	5.127 110
0.001	5.012 521
0.0001	5.001 250

t	$f(t)$
–0.5	1.835 830
–0.1	3.934 693
–0.01	4.877 058
–0.001	4.987 521
–0.0001	4.998 750

It appears that $\lim_{t\to 0}\dfrac{e^{5t}-1}{t}=5$.

30. For $f(t)=\dfrac{(2+h)^5-32}{h}$:

h	$f(h)$
0.5	131.312 500
0.1	88.410 100
0.01	80.804 010
0.001	80.080 040
0.0001	80.008 000

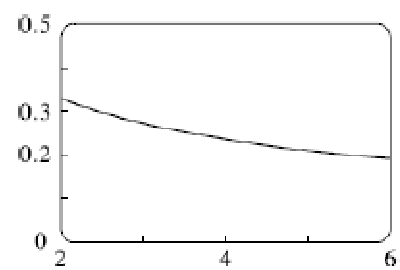
h	$f(h)$
–0.5	48.812 500
–0.1	72.390 199
–0.01	79.203 990
–0.001	79.920 040
–0.0001	79.992 000

It appears that $\lim_{h\to 0}\dfrac{(2+h)^5-32}{h}=80$.

31. For $f(x)=\dfrac{\ln x-\ln 4}{x-4}$:

x	$f(x)$
3.9	0.253 178
3.99	0.250 313
3.999	0.250 031
3.9999	0.250 003

x	$f(x)$
4.1	0.246 926
4.01	0.249 688
4.001	0.249 969
4.0001	0.249 997

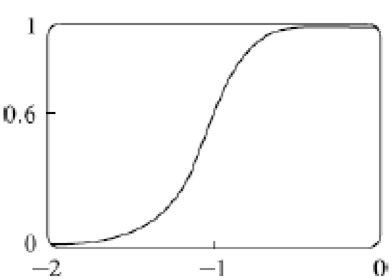


It appears that $\lim_{x\to 4}\dfrac{\ln x-\ln 4}{x-4}=0.25$. The graph confirms that result.

32. For $f(p)=\dfrac{1+p^9}{1+p^{15}}$:

p	$f(p)$
–1.1	0.427 397
–1.01	0.582 008
–1.001	0.598 200
–1.0001	0.599 820

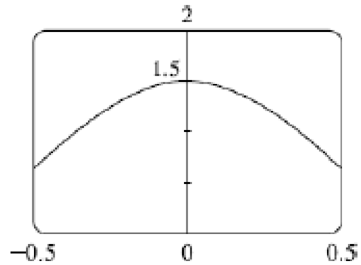
p	$f(p)$
–0.9	0.771 405
–0.99	0.617 992
–0.999	0.601 800
–0.9999	0.600 180



It appears that $\lim_{p\to -1}\dfrac{1+p^9}{1+p^{15}}=0.6$. The graph confirms that result.

33. For $f(\theta) = \dfrac{\sin 3\theta}{\tan 2\theta}$:

θ	$f(\theta)$
±0.1	1.457 847
±0.01	1.499 575
±0.001	1.499 575
±0.0001	1.499 9960

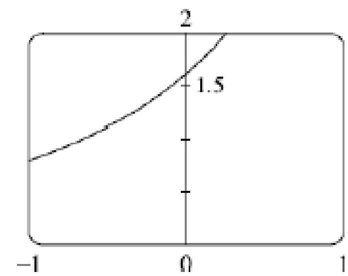


It appears that $\lim_{\theta \to 0} \dfrac{\sin 3\theta}{\tan 2\theta} = 1.5$. The graph confirms that result.

34. For $f(t) = \dfrac{5^t - 1}{t}$:

t	$f(t)$
0.1	1.746 189
0.01	1.622 459
0.001	1.622 459
0.0001	1.610 734

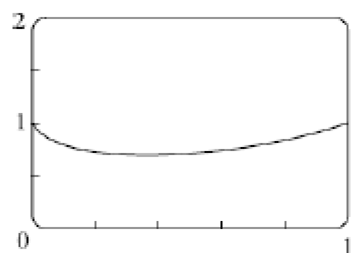
t	$f(t)$
−0.1	1.486 601
−0.01	1.596 556
−0.001	1.622 459
−0.0001	1.610 734



It appears that $\lim_{t \to 0} \dfrac{5^t - 1}{t} \approx 1.6094$ (which is $\ln 5$). The graph confirms that result.

35. For $f(x) = x^x$:

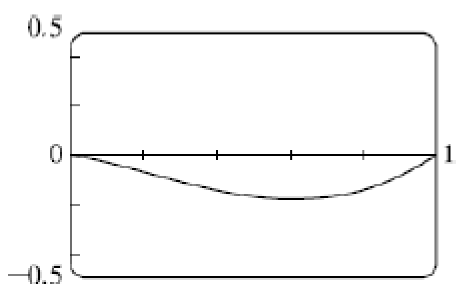
x	$f(x)$
0.1	0.794 328
0.01	0.954 993
0.001	0.993 116
0.0001	0.999 079



It appears that $\lim_{x \to 0^+} x^x = 1$. The graph confirms that result.

36. For $f(x) = x^2 \ln x$:

x	$f(x)$
0.1	−0.023 026
0.01	−0.000461
0.001	−0.000 007
0.0001	−0.000 000



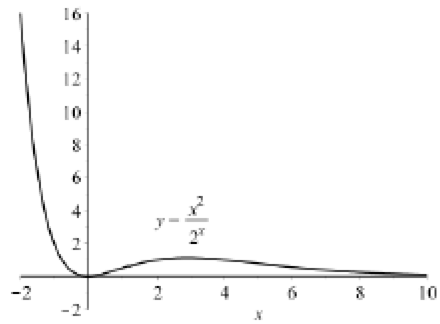
It appears that $\lim_{x \to 0^+} x^2 \ln x = 0$. The graph confirms that result.

p. 129: 17 (a-d), 25, 27, 29-35, 45-52

17. (a) $\lim_{x\to\infty} f(x) = -2$ (b) $\lim_{x\to-\infty} f(x) = 2$ (c) $\lim_{x\to 1} f(x) = \infty$

(d) $\lim_{x\to 3} f(x) = -\infty$

25. If $f(x) = x^2/2^x$, then a calculator gives $f(0)=0, f(1)=0.5, f(2)=1, f(3)=1.125, f(4)=1,$ $f(6)=0.5625,\ f(5)=0.78125,\ f(7)=0.3828125,\ f(8)=0.25,\ f(9)=0.158203125,$ $f(10)=0.09765625,\ f(20)\approx 0.00038147,\ f(50)\approx 2.2204\times 10^{-12},\ f(100)\approx 7.8886\times 10^{-27}.$ Combining this information with a graph, it appears that $\lim_{x\to\infty}\left(x^2/2^x\right)=0.$



27.
$$\lim_{x\to\infty}\frac{2x^2-7}{5x^2+x-3}=\lim_{x\to\infty}\frac{\left(2x^2-7\right)/x^2}{\left(5x^2+x-3\right)/x^2}$$
$$=\frac{\lim_{x\to\infty}\left(2-\frac{7}{x^2}\right)}{\lim_{x\to\infty}\left(5+\frac{1}{x}-\frac{3}{x^2}\right)}$$
$$=\frac{2-7\lim_{x\to\infty}\left(\frac{1}{x^2}\right)}{5+\lim_{x\to\infty}\left(\frac{1}{x}\right)-3\lim_{x\to\infty}\left(\frac{1}{x^2}\right)}$$
$$=\lim_{x\to\infty}\frac{\left(2-\frac{7}{x^2}\right)}{\left(5+\frac{1}{x}-\frac{3}{x^2}\right)}$$
$$=\frac{2-7(0)}{5+0+3(0)}=\frac{2}{5}$$

29. $$\lim_{x\to\infty}\frac{3x-2}{2x+1}=\lim_{x\to\infty}\frac{3-\frac{2}{x}}{2+\frac{1}{x}}=\frac{3-2(0)}{2+0}=\frac{3}{2}$$

30. $$\lim_{x\to\infty}\frac{1-x^2}{x^3-x+1}=\lim_{x\to\infty}\frac{\frac{1}{x^3}-\frac{1}{x}}{1-\frac{1}{x^2}+\frac{1}{x^3}}=\frac{0-2(0)}{1-0+0}=0$$

31. $$\lim_{x\to-\infty}\frac{x-2}{x^2+1}=\lim_{x\to-\infty}\frac{\frac{1}{x}-\frac{2}{x^2}}{1+\frac{1}{x^2}}=\frac{0-2(0)}{1+0}=0$$

32. $$\lim_{x\to-\infty}\frac{4x^3+6x^2-2}{2x^3-4x+5}=\lim_{x\to-\infty}\frac{4+\frac{6}{x}-\frac{2}{x^3}}{2-\frac{4}{x^2}+\frac{5}{x^3}}=\frac{4-0-0}{2-0+0}=2$$

33. $\lim_{t\to\infty}\frac{\sqrt{t}+t^2}{2t-t^2}=\lim_{t\to\infty}\frac{\left(\sqrt{t}+t^2\right)/t^2}{\left(2t-t^2\right)/t^2}=\lim_{t\to\infty}\frac{\frac{1}{t^{3/2}}+1}{\left(\frac{2}{t}-1\right)}=\frac{0+1}{0-1}=-1$

34. $\lim_{t\to\infty}\frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}=\lim_{t\to\infty}\frac{\frac{1}{t^{1/2}}-1}{2+3\frac{1}{t^{1/2}}-5\frac{1}{t^{3/2}}}=\frac{0-1}{2+0-0}=-\frac{1}{2}$

35. $\lim_{x\to\infty}\frac{\left(2x^2+1\right)^2}{(x-1)^2\left(x^2+x\right)}=\lim_{x\to\infty}\frac{\left(2x^2+1\right)^2/x^4}{(x-1)^2\left(x^2+x\right)/x^4}=\lim_{x\to\infty}\frac{\left(2+\frac{1}{x^2}\right)^2}{\left[\left(x^2-2x+1\right)/x^2\right]\left[\left(x^2+x\right)/x^2\right]}$

$$=\lim_{x\to\infty}\frac{\left(2+\frac{1}{x^2}\right)^2}{\left[\left(1-\frac{2}{x}+\frac{1}{x^2}\right)\right]\left[\left(1+\frac{1}{x}\right)\right]}=\frac{(2+0)^2}{(1-0+0)(1+0)}=4$$

45. $\lim_{x\to\infty}\frac{x^4-3x^2+x}{x^3-x+2}=\lim_{x\to\infty}\frac{x-\frac{3}{x}+\frac{1}{x^2}}{1-\frac{1}{x^2}+\frac{2}{x^3}}=\infty$ since the numerator increases without bound and the denominator approaches 1 as $x\to\infty$.

46. $\lim_{x\to\infty}\left(e^{-x}+2\cos 3x\right)$ does not exist. $\lim_{x\to\infty}e^{-x}=0$, but $\lim_{x\to\infty}(2\cos 3x)$ does not exist because the values of $2\cos 3x$ oscillate between the values of –2 and 2 infinitely often.

47. $\lim_{x\to-\infty}\left(x^2+2x^7\right)=\lim_{x\to-\infty}x^7\left(\frac{1}{x^5}+2\right)=-\infty$ because $x^7\to-\infty$ and $\frac{1}{x^5}+2\to 2$ as $x\to-\infty$.

48. $\lim_{x\to-\infty}\frac{1+x^6}{x^4+1}=\lim_{x\to-\infty}\frac{\frac{1}{x^4}+x^2}{1+\frac{1}{x^4}}=\infty$ since the numerator increases without bound and the denominator approaches 1 as $x\to\infty$.

49. Let $t=e^x$. As $x\to\infty, t\to\infty$. Then $\lim_{x\to\infty}\arctan\left(e^x\right)=\lim_{t\to\infty}(\arctan t)=\frac{\pi}{2}$.

50. Divide the numerator and denominator by e^{3x} : $\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}=\lim_{x\to\infty}\frac{1-e^{-6x}}{1+e^{-6x}}=\frac{1-0}{1+0}=1$

51. $\lim_{x\to\infty}\frac{1-e^x}{1+2e^x}=\lim_{x\to\infty}\frac{\frac{1}{e^x}-1}{\frac{1}{e^x}+2}=\frac{0-1}{0+2}=-\frac{1}{2}$

52. Since $0\le\sin^2 x\le 1$, we have $0\le\frac{\sin^2 x}{x^2+1}\le\frac{1}{x^2+1}$. We know that $\lim_{x\to\infty}0=0$ and $\lim_{x\to\infty}\frac{1}{x^2+1}=0$, so by the Squeeze Theorem, $\lim_{x\to\infty}\frac{\sin^2 x}{x^2+1}=0$.