Limits, Part 2

p: 102: 7-8, 10, 11-12 (skip f), 14-15, 39-51

7. (a) As x approaches 2 from the left, the values of $f(x)$ approach 3, so $\lim_{x\to 2^-} f(x) = 3$.

   (b) As x approaches 2 from the right, the values of $f(x)$ approach 1, so $\lim_{x\to 2^+} f(x) = 1$.

   (c) $\lim_{x\to 2} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

   (d) When $x = 2$, $y = 3$, so $f(2) = 3$.

   (e) As x approaches 4, the values of $f(x)$ approach 4, so $\lim_{x\to 4} f(x) = 4$.

   (f) $f(4) = 4$

8. (a) As x approaches 1, the values of $f(x)$ approach 1.7, so $\lim_{x\to 1} f(x) = 1.7$.

   (b) As x approaches 3 from the left, the values of $f(x)$ approach 1, so $\lim_{x\to 3^-} f(x) = 1$.

   (c) As x approaches 3 from the right, the values of $f(x)$ approach 4, so $\lim_{x\to 3^+} f(x) = 4$.

   (d) $\lim_{x\to 3} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

   (e) When $x = 3$, $y = 3$, so $f(3) = 3$.

10. (a) $\lim_{x\to 0^-} g(x) = -1$ (b) $\lim_{x\to 0^+} g(x) = -2$

    (c) $\lim_{x\to 0} g(x)$ does not exist because the limits in part (a) and part (b) are not equal.

    (d) $\lim_{x\to 2^-} g(x) = 2$ (e) $\lim_{x\to 2^+} g(x) = 0$

    (f) $\lim_{x\to 2} g(x)$ does not exist because the limits in part (d) and part (e) are not equal.

    (g) $g(2) = 1$ (h) $\lim_{x\to 4} g(x) = 3$

11. (a) $\lim_{x\to -3} f(x) = \infty$ (b) $\lim_{x\to 2^-} f(x) = -\infty$ (c) $\lim_{x\to 2^+} f(x) = \infty$

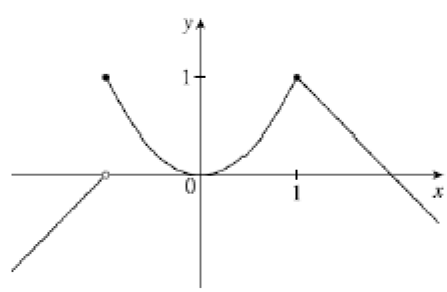
    (d) $\lim_{x\to -1} f(x) = -\infty$

12. (a) $\lim_{x\to -7} f(x) = -\infty$ (b) $\lim_{x\to -3} f(x) = \infty$ (c) $\lim_{x\to 0} f(x) = \infty$

    (d) $\lim_{x\to 6^-} f(x) = -\infty$ (e) $\lim_{x\to 6^+} f(x) = \infty$

14. From the graph of

$$f(x) = \begin{cases} 1+x, & \text{if } x < -1 \\ x^2, & \text{if } -1 \le x < 1, \\ 2-x, & \text{if } x \ge 1 \end{cases}$$



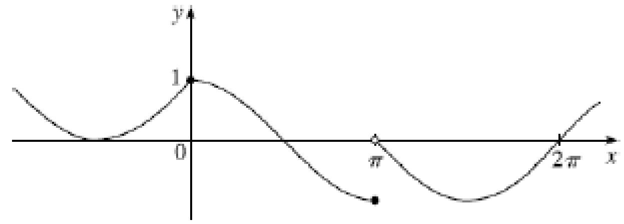
(a) $\lim_{x\to -1^-} f(x) = 0$ (b) $\lim_{x\to -1^+} f(x) = 1$

(c) $\lim_{x\to -1} f(x)$ does not exist because the limits in part (a) and part (b) are not equal.

(d) $\lim_{x\to 1^-} f(x) = 1$ (e) $\lim_{x\to 1^+} f(x) = 1$ (f) $\lim_{x\to 1} f(x) = 1$

15. From the graph of

$$f(x)=\begin{cases}1+\sin x, & \text{if } x<0\\ \cos x, & \text{if } 0\le x\le \pi,\\ \sin x, & \text{if } x>\pi\end{cases}$$



(a) $\lim_{x\to 0^-} f(x)=1$ (b) $\lim_{x\to 0^+} f(x)=1$

(c) $\lim_{x\to 0} f(x)=1$ (d) $\lim_{x\to \pi^-} f(x)=-1$

(e) $\lim_{x\to \pi^+} f(x)=0$ (f) $\lim_{x\to \pi} f(x)=0$ does not exist because the limits in part (d) and part (e) are not equal.

39. $\lim_{x\to 5^+}\dfrac{x+1}{x-5}=\infty$ since the numerator is positive and the denominator approaches 0 from the positive size as $x\to 5^+$.

40. $\lim_{x\to 5^-}\dfrac{x+1}{x-5}=-\infty$ since the numerator is positive and the denominator approaches 0 from the negative size as $x\to 5^-$.

41. $\lim_{x\to 1}\dfrac{2-x}{(x-1)^2}=\infty$ since the numerator is positive and the denominator approaches 0 through positive values as $x\to 1$.

42. $\lim_{x\to 3^-}\dfrac{\sqrt{x}}{(x-3)^2}=\dfrac{3}{+0}=\infty$

43. Let $t=x^2-9$. Then as $x\to 3^+, t\to 0^+$, and $\lim_{x\to 3^+}\ln\left(x^2-9\right)=\lim_{t\to 0^+}\ln t=-\infty$.

44. $\lim_{x\to 0^+}\ln(\sin x)=-\infty$ since $\sin x\to 0^+$ as $x\to 0^+$.

45. $\lim_{x\to(\pi/2)^+}\dfrac{1}{x}\sec x=-\infty$ since $\dfrac{1}{x}$ is positive and $\sec x\to-\infty$ as $x\to(\pi/2)^+$.

46. $\lim_{x\to\pi^-}\cot x=\lim_{x\to\pi^-}\dfrac{\cos x}{\sin x}=-\infty$ since the numerator is negative and the denominator approaches 0 through positive values as $x\to\pi^-$.

47. $\lim_{x\to 2\pi^-}x\csc x=\lim_{x\to 2\pi^-}\dfrac{x}{\sin x}=-\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x\to 2\pi^-$.

48. $\lim_{x\to 2^-}\dfrac{x^2-2x}{x^2-4x+4}=\lim_{x\to 2^-}\dfrac{x(x-2)}{(x-2)^2}=\lim_{x\to 2^-}\dfrac{x}{x-2}=-\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x\to 2^-$.

49. $\lim_{x\to 2^+}\dfrac{x^2-2x-8}{x^2-5x+6}=\lim_{x\to 2^+}\dfrac{(x-4)(x+2)}{(x-3)(x-2)}=\infty$ since the numerator is negative and the denominator approaches 0 through negative values as $x\to 2^+$.

50. $\lim_{x\to 0^+}\left(\dfrac{1}{x}-\ln x\right)=\infty$ since $\dfrac{1}{x}\to\infty$ and $\ln x\to-\infty$ as $x\to 0^+$.

51. $\lim_{x\to 0}\left(\ln x^2-x^{-2}\right)=-\infty$ since $\ln x^2\to-\infty$ and $x^{-2}\to\infty$ as $x\to 0$.