p. 487: 7-8, 15-16, 21-31 odd, 32, 40-44, 61, 73

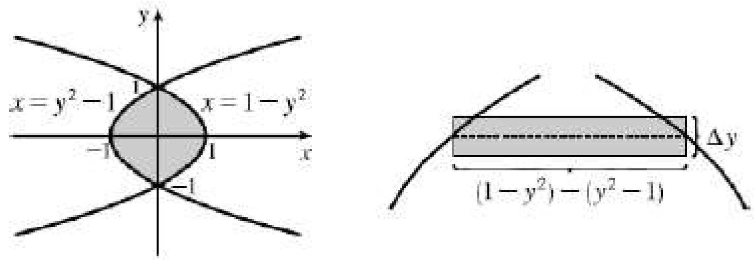
7. $A=\int_{y=-1}^{y=1}(x_R-x_L)\,dy=\int_{-1}^{1}\left[e^y-(y^2-2)\right]dy=\int_{-1}^{1}(e^y-y^2+2)\,dy$

$=\left[e^y-\frac{1}{3}y^3+2y\right]_{-1}^{1}=(e^1-\frac{1}{3}+2)-(e^{-1}+\frac{1}{3}-2)=e-\frac{1}{e}+\frac{10}{3}$

8. $A=\int_0^3\left[\left(2y-y^2\right)-\left(y^2-4y\right)\right]dy=\int_0^3(-2y^2+6y)\,dy=\left[-\frac{2}{3}y^3+3y^2\right]_0^3=(-18+27)-0=9$

15. The curves intersect when $1-y^2=y^2-1\Leftrightarrow 2=2y^2\Leftrightarrow y^2=1\Leftrightarrow y=\pm1$.

$A=\int_{-1}^{1}\left[(1-y^2)-(y^2-1)\right]dy=\int_{-1}^{1}2(1-y^2)\,dy=2\cdot2\int_0^1(1-y^2)\,dy=4\left[y-\frac{1}{3}y^3\right]_0^1=4\left(1-\frac{1}{3}\right)=\frac{8}{3}$

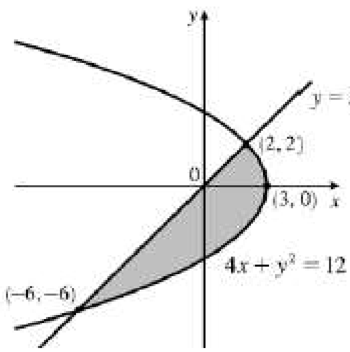


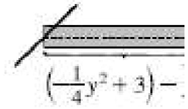
16. $4x+x^2=12\Leftrightarrow(x+6)(x-2)=0\Leftrightarrow$
$x=-6$ or $x=2$, so $y=-6$ or $y=2$, and

$A=\int_{-6}^{2}\left[(-\frac{1}{4}y^2+3)-y\right]dy$

$=\left[-\frac{1}{12}y^3-\frac{1}{2}y^2+3y\right]_{-6}^{2}$

$=\left(-\frac{2}{3}-2+6\right)-\left(18-18-18\right)=22-\frac{2}{3}=\frac{64}{3}$



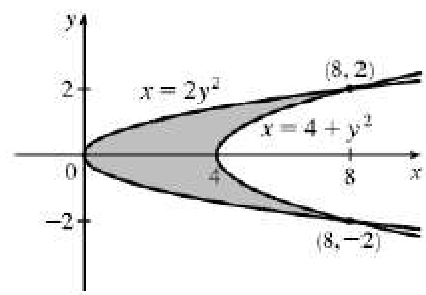


21. $2y^2=4+y^2\Leftrightarrow y^2=4\Leftrightarrow y=\pm2$, so

$A=\int_{-2}^{2}\left[(4+y^2)-2y^2\right]dy$

$=2\int_0^2(4-y^2)\,dy$ [by symmetry]

$=2\left[4y-\frac{1}{3}y^3\right]_0^2=2\left(8-\frac{8}{3}\right)=\frac{32}{3}$

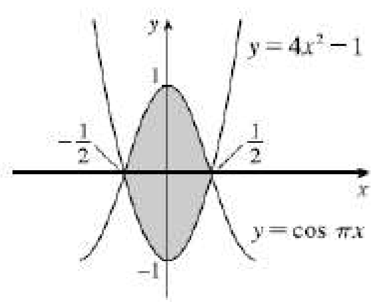


23. By inspection, the curves intersect at $x=\pm\frac{1}{2}$.

$A=\int_{-1/2}^{1/2}\left[\cos\pi x-(4x^2-1)\right]dx$

$=2\int_0^{1/2}\left(\cos\pi x-4x^2+1\right)dx$ [by symmetry]

$=2\left[\frac{1}{\pi}\sin\pi x-\frac{4}{3}x^3+x\right]_0^{1/2}=2\left[\left(\frac{1}{\pi}-\frac{1}{6}+\frac{1}{2}\right)-0\right]=2\left(\frac{1}{\pi}+\frac{1}{3}\right)=\frac{2}{\pi}+\frac{2}{3}$

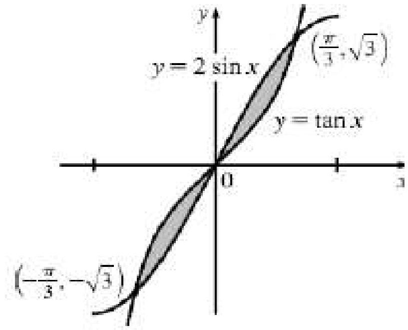


25. The curves intersect when $\tan x = 2\sin x$ $\left(\text{on } [-\pi/3, \pi/3]\right) \Leftrightarrow$
$\sin x = 2\sin x \cos x \Leftrightarrow 2\sin x\cos x - \sin x = 0 \Leftrightarrow$
$\sin x(2\cos x - 1) = 0 \Leftrightarrow \sin x = 0$ or $\cos x = \frac{1}{2} \Leftrightarrow x = 0$ or $x = \pm\frac{\pi}{3}$.

$A = 2\int_0^{\pi/3}(2\sin x - \tan x)\,dx$ [by symmetry]

$= 2\left[-2\cos x - \ln|\sec x|\right]_0^{\pi/3} = 2\left[(-1-\ln 2) - (-2-0)\right]$

$= 2(1-\ln 2) = 2 - 2\ln 2$



27. The curves intersect when

$\sqrt[3]{2x} = \frac{1}{8}x^2 \Leftrightarrow 2x = \dfrac{1}{(2^3)^3}x^6 \Leftrightarrow 2^{10}x = x^6 \Leftrightarrow x^6 - 2^{10}x = 0 \Leftrightarrow$

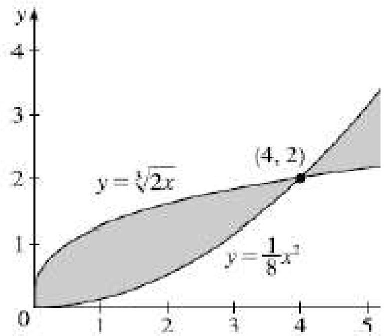
$x\left(x^5 - 2^{10}\right) = 0 \Leftrightarrow x = 0$ or $x^5 = 2^{10}$ $x = 0$ or $x = 2^2 = 4$, so for $0 \le x \le 6$,

$A = \int_0^4\left(\sqrt[3]{2x} - \frac{1}{8}x^2\right)dx + \int_4^5\left(\frac{1}{8}x^2 - \sqrt[3]{2x}\right)dx$

$= \left[\frac{3}{4}\sqrt[3]{2}x^{4/3} - \frac{1}{24}x^3\right]_0^4 + \left[\frac{1}{24}x^3 - \frac{3}{4}\sqrt[3]{2}x^{4/3}\right]_4^6$

$= \left(\frac{3}{4}\sqrt[3]{2}\cdot 4\sqrt[3]{4} - \frac{64}{24}\right) - (0-0) + \left(\frac{216}{24} - \frac{3}{4}\sqrt[3]{2}\cdot 6\sqrt[3]{6}\right) - \left(\frac{64}{24} - \frac{3}{4}\sqrt[3]{2}\cdot 4\sqrt[3]{4}\right)$

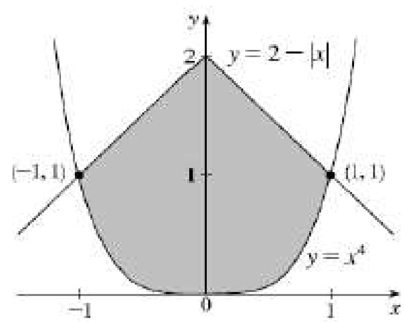
$= 6 - \frac{8}{3} + 9 - \frac{9}{2}\sqrt[3]{12} - \frac{8}{3} + 6 = \frac{47}{3} - \frac{9}{2}\sqrt[3]{12}$



29. By inspection, we see that the curves intersect at $x = \pm 1$ and that the area of the region enclosed by the curves is twice the area enclosed in the first quadrant.

$A = 2\int_0^1\left[(2-x) - x^4\right]dx = 2\left[2x - \frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1$

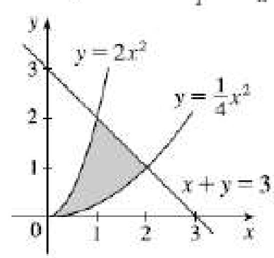
$= 2\left[\left(2 - \frac{1}{2} - \frac{1}{5}\right) - 0\right] = 2\left(\frac{13}{10}\right) = \frac{13}{5}$



31. $\frac{1}{4}x^2 = -x + 3 \Leftrightarrow x^2 + 4x - 12 = 0 \Leftrightarrow (x+6)(x-2) = 0 \Leftrightarrow x = -6$ or 2 and
$2x^2 = -x + 3 \Leftrightarrow 2x^2 + x - 3 = 0 \Leftrightarrow (2x+3)(x-1) = 0 \Leftrightarrow x = -\frac{3}{2}$ or 1,
for $x \ge 1$.

$A = \int_0^1(2x^2 - \frac{1}{4}x^2)\,dx + \int_1^2\left[(-x+3) - \frac{1}{4}x^2\right]dx = \int_0^1 \frac{7}{4}x^2\,dx + \int_1^2\left[-\frac{1}{4}x^2 - x + 3\right]dx$

$= \frac{7}{12}x^3\Big]_0^1 + \left[-\frac{1}{12}x^3 - \frac{1}{2}x^2 + 3x\right]_1^2 = \frac{7}{12} + \left(-\frac{2}{3} - 2 + 6\right) - \left(-\frac{1}{12} - \frac{1}{2} + 3\right) = \frac{3}{2}$



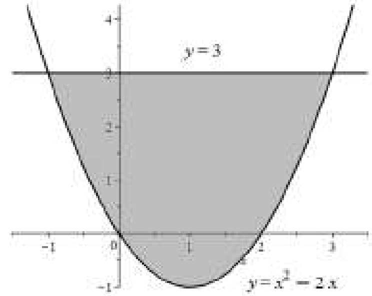
32. (a) Total area $= 12 + 27 = 39$.
(b) $f(x) \le g(x)$ for $0 \le x \le 2$ and $f(x) \ge g(x)$ for $2 \le x \le 5$, so

$$\int_0^5[f(x) - g(x)]dx = \int_0^2[f(x) - g(x)]dx + \int_2^5[f(x) - g(x)]dx = -\int_0^2[g(x) - f(x)]dx + \int_2^5[f(x) - g(x)]d$$

$$= -(12) + 27 = 15.$$

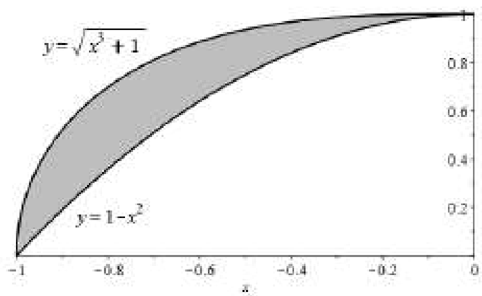
40. The curves intersect at $x=-1$ and $x=3$, so the area of the region bound by the curves is

$$A=\int_{-1}^{3}\left[3-(x^2-2x)\right]dx$$
$$=\left[3x-\tfrac{1}{3}x^3+x^2\right]_{-1}^{3}$$
$$=(9-9+9)-\left(-3+\tfrac{1}{3}+1\right)=\tfrac{32}{3}, \text{ option } (\mathbf{B}).$$



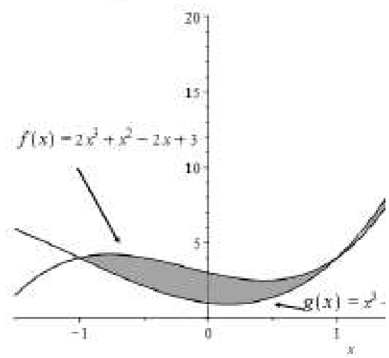
41. The curves intersect at $x=-1$ and $x=0$, so the area of the region bound by these curves is

$$A=\int_{-1}^{0}\left[\sqrt{x^3+1}-(1-x^2)\right]dx=0.1746, \text{ option } (\mathbf{A}).$$



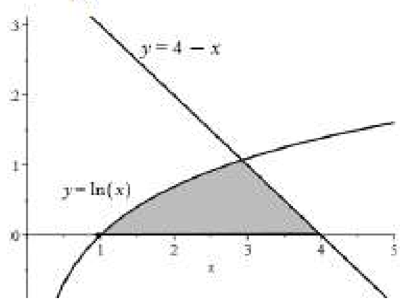
42. Solving, we find that the curves intersect at $x=-1, x=1$ and $x=2$. The area of the region bounded by these curves is

$$A=\int_{-1}^{1}[f(x)-g(x)]dx+\int_{1}^{2}[g(x)_{f}-g(x)]dx$$
$$=\int_{-1}^{1}(x^3-2x^2-x+2)\,dx+\int_{1}^{2}(-x^3+2x^2+x-2)dx$$
$$=\left[\tfrac{1}{4}x^4-\tfrac{2}{3}x^3-\tfrac{1}{2}x^2+2x\right]_{-1}^{1}+\left[-\tfrac{1}{4}x^4+\tfrac{2}{3}x^3+\tfrac{1}{2}x^2-2x\right]_{1}^{2}$$

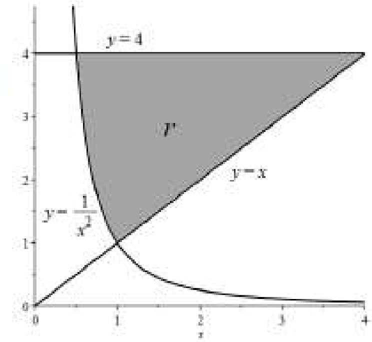


$$=\left(\tfrac{1}{4}-\tfrac{2}{3}-\tfrac{1}{2}+2\right)-\left(\tfrac{1}{4}+\tfrac{2}{3}-\tfrac{1}{2}-2\right)+\left(-4+\tfrac{16}{3}+2-4\right)-\left(-\tfrac{1}{4}+\tfrac{2}{3}+\tfrac{1}{2}-2\right)=\tfrac{37}{12}\approx 3.084, \text{ option } (\mathbf{C}).$$

43. Using technology, we find that the curves intersect at $x=a\approx 2.92627$, so the area bounded by the axis and these curves is $A=\int_{1}^{a}\ln x\,dx+\int_{a}^{4}(4-x)\,dx\approx 1.792$, option (**B**).



44. From the graph, we see that the curves $y=\dfrac{1}{x^2}$ and $y=4$ intersect at $\left(\frac{1}{2},4\right)$, the curves $y=\dfrac{1}{x^2}$ and $y=x$ intersect at the point $(1,1)$, and the curves $y=x$ and $y=4$ intersect at the point $(4,4)$. Thus, the area of the region bounded by these curves is



$$A=\int_{1/2}^{1}\left(4-\frac{1}{x^2}\right)dx+\int_1^4(4-x)\,dx=\left[4x+\frac{1}{x}\right]_{1/2}^{1}+\left[4x-\tfrac{1}{2}x^2\right]_1^4$$
$$=(4+1)-(2+2)+(16-8)-\left(4-\tfrac{1}{2}\right)=\tfrac{11}{2}$$

61. (a) At time $t=2$ seconds, Adam's velocity is $v_A(2)=\dfrac{40(2)}{\sqrt{2^2+20}}=\dfrac{80}{\sqrt{24}}\approx 16.2399$ ft/s, and Bassam's velocity is $v_B(2)=\dfrac{36(2)}{\sqrt{2^2+10}}=\dfrac{72}{\sqrt{14}}\approx 19.243$ ft/s, so Bassam is traveling approximately 2.913 ft/s faster than Adam.

(b) The bicyclists have equal velocity when $v_A(t)=v_B(t)\Leftrightarrow \dfrac{40t}{\sqrt{t^2+20}}=\dfrac{36t}{\sqrt{t^2+10}}\Leftrightarrow$

$\dfrac{\sqrt{t^2+10}}{\sqrt{t^2+20}}=\dfrac{36}{40}=0.9\Leftrightarrow\dfrac{t^2+10}{t^2+20}=0.81\Leftrightarrow t^2+10=0.81(t^2+20)\Leftrightarrow 0.19t^2=6.2\Leftrightarrow t^2=\frac{620}{19}\Leftrightarrow$

$t=\sqrt{\frac{620}{19}}\approx 5.712$ seconds $[t>0]$.

(c) We know that the area under the graph of $v_A(t)$ between $t=0$ and $t=20$ is $\int_0^{20}v_A(t)\,dt=s_A(20)$, gives Adam's displacement after 20 seconds. Similarly, the area under the graph of $v_B(t)$ between $t=0$ and $t=20$ is $\int_0^{20}v_B(t)\,dt=s_B(20)$ gives Bassam's displacement after 20 seconds. Then

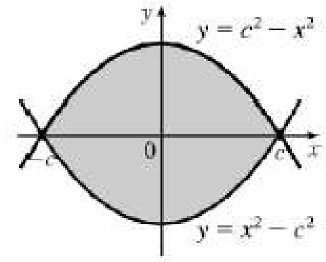
$s_A(20)=\displaystyle\int_0^{20}\left(\frac{40t}{\sqrt{t^2+20}}\right)dt=\left[40(t^2+20)^{1/2}\right]_0^{20}=40\left(\sqrt{420}-\sqrt{20}\right)\approx 640.871$ ft, and

$s_b(20)=\displaystyle\int_0^{20}\left(\frac{36t}{\sqrt{t^2+10}}\right)dt=\left[30(t^2+10)^{1/2}\right]_0^{20}=36\left(\sqrt{410}-\sqrt{10}\right)\approx 615.102$ ft. Thus, Adam is roughly 26 feet ahead of Bassam.

(d) $d_A(t)=\displaystyle\int_0^{t}\left(\frac{40x}{\sqrt{x^2+20}}\right)dx=\left[40(x^2+20)^{1/2}\right]_0^{t}=40\sqrt{t^2+20}-40\sqrt{20}$, and

$d_B(t)=\displaystyle\int_0^{t}\left(\frac{36x}{\sqrt{x^2+10}}\right)dx=\left[36(x^2+10)^{1/2}\right]_0^{t}=36\sqrt{t^2+10}-36\sqrt{20}.$

73. We first assume that $c > 0$, since c can be replaced by $-c$ in both equations without changing the graphs, and if $c = 0$ the curves do not enclose a region. We see from the graph that the enclosed area A lies between $x = -c$ and $x = c$, and by symmetry, it is equal to four times the area in the first quadrant. The enclosed area is



$$A = 4\int_0^c \left(c^2 - x^2\right) dx = 4\left[c^2 x - \tfrac{1}{3}x^3\right]_0^c = 4\left(c^3 - \tfrac{1}{3}c^3\right) = 4\left(\tfrac{2}{3}c^3\right) = \tfrac{8}{3}c^3.$$

So $A = 576 \Leftrightarrow \tfrac{8}{3}c^3 = 576 \Leftrightarrow c^3 = 216 \Leftrightarrow c = \sqrt[3]{216} = 6.$

Note that is $c = -6$ is another solution, since the graphs are the same.

p. 494: 7-15 odd, 20-21, 23-24

7. $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{1}{4}\left(\frac{2}{3}\cdot 8\right) = \frac{4}{3}$

9. $g_{\text{ave}} = \frac{1}{b-a}\int_a^b g(t)\,dt = \frac{1}{3-1}\int^{2} \frac{t}{\sqrt{3+t^2}}\,dt = \frac{1}{2}\left[(3+t^2)^{1/2}\right]_1^3 = \frac{1}{2}\left(2\sqrt{3}-2\right) = \sqrt{3}-1$

11. $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{1-(-1)}\int_{-1}^{1} \frac{x^2}{(x^3+3)^2}\,dx = \frac{1}{2}\int_2^4 \frac{1}{u^2}\cdot\left(\frac{1}{3}du\right) \quad \left[\begin{array}{c} u = x^3+3 \\ du = 3x^2\,dx \end{array}\right] = \frac{1}{6}\left[-\frac{1}{u}\right]_2^4 = \frac{1}{6}\left(-\frac{1}{4}+\frac{1}{2}\right) = \frac{1}{24}$

13. $h_{\text{ave}} = \frac{1}{b-a}\int_a^b h(u)\,du = \frac{1}{5-1}\int^{5} \frac{\ln u}{u}\,du = \frac{1}{4}\int_0^{\ln 5} y\,dy \quad \left[\begin{array}{c} y = \ln u \\ dy = 1/u\,du \end{array}\right] = \frac{1}{4}\left[\frac{1}{2}y^2\right]_0^{\ln 5} = \frac{1}{8}(\ln 5)^2$

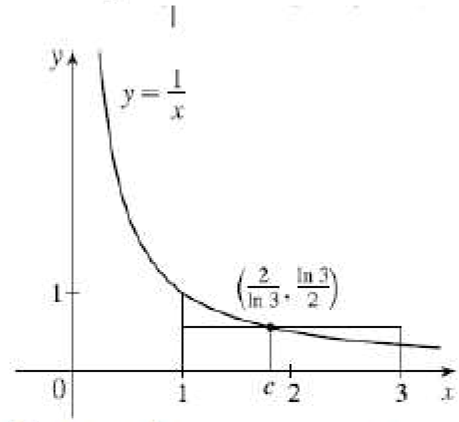
15. (a) $f_{\text{ave}} = \frac{1}{3-1}\int^{2} \frac{1}{x}\,dx = \frac{1}{2}\left[\ln|x|\right]_1^3$

$= \frac{1}{2}\left[\ln 3 - \ln 1\right] = \frac{1}{2}\ln 3$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow \frac{1}{c} = \frac{1}{2}\ln 3 \Leftrightarrow$

$c = \frac{2}{\ln 3} \approx 1.820.$

(c)



20. $f_{\text{ave}} = \frac{1}{4-1}\int_1^4 f(x)\,dx = \frac{1}{3}\int_1^2 x^2\,dx + \frac{1}{3}\int_2^4 2x\,dx = \frac{1}{3}\left[\frac{1}{3}x^3\right]_1^2 + \frac{1}{3}\left[x^2\right]_2^4 = \frac{1}{9}(8-1) + \frac{1}{3}(16-4) = \frac{7}{9}+4 = \frac{43}{9}$

21. Use geometric interpretations to find the values of the integrals.

$$\int_0^8 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx + \int_4^6 f(x)\,dx + \int_6^7 f(x)\,dx + \int_7^8 f(x)\,dx$$

$$= -\tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{2} + 1 + 4 + \tfrac{3}{2} + 2 = 9$$

Thus, the average value of f on $[0,8] = f_{\text{ave}} = \frac{1}{8-0}\int_0^8 f(x)\,dx = \frac{1}{8}(9) = \frac{9}{8}$

23. $f_{\text{ave}} = \frac{1}{2-(-1)}\int_{-1}^{2}\left[3x^2+2x\right]dx = \frac{1}{3}\left[x^3+x^2\right]_{-1}^{2} = \frac{1}{3}(8+4) - \frac{1}{3}(-1+1) = 4$, option (**A**).

24. The average value of a function over the interval $[-1,1]$ is $f_{\text{ave}} = \frac{1}{1-(-1)} \int_{-1}^{1} f(x)\,dx = \frac{1}{2}\int_{-1}^{1} f(x)\,dx$.

For $f(x) = x^3$, $f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} x^3\,dx = \frac{1}{2}\cdot\frac{1}{2}x^4\Big]_{-1}^{1} = \frac{1}{8}((1-1) = 0$.

For $f(x) = \sin x$, $f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} \sin x\,dx = -\frac{1}{2}\cos x\Big]_{-1}^{1} = -\frac{1}{2}(\cos 1 - \cos(-1)) = 0$.

For $f(x) = xe^{x^2}$, $f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} xe^{x^2}\,dx \quad \begin{bmatrix} u = x^2, \\ \frac{1}{2}\,du = x\,dx \end{bmatrix} \quad = \frac{1}{2}\cdot\frac{1}{2}e^{x^2}\Big]_{-1}^{1} = \frac{1}{4}\left(e^{1-e^{1}}\right) = 0$.

But for **(B)**, $f(x) = 3x^2$, $f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} 3x^2\,dx = \frac{1}{2}x^3\Big]_{-1}^{1} = \frac{1}{2}\big(1-(-1)\big) = 1 \neq 0$.

p. 538: 9-15 odd, 21, 31

9. $y = \sin x \Rightarrow dy/dx = \cos x \Rightarrow 1 + (dy/dx)^2 = 1 + \cos^2 x$. So $L = \int_0^{\pi} \sqrt{1+\cos^2 x}\,dx \approx 3.8202$.

11. $y = x - \ln x \Rightarrow dy/dx = 1 - 1/x \Rightarrow 1 + (dy/dx)^2 = 1 + (1-1/x)^2$. So $L = \int_1^4 \sqrt{1+(1-1/x)^2}\,dx \approx 3.4467$.

13. $x = \sqrt{y} - y \Rightarrow dx/dy = 1/\left(2\sqrt{y}\right) - 1 \Rightarrow 1 + (dx/dy)^2 = 1 + \left(\frac{1}{2\sqrt{y}} - 1\right)^2$.

So $L = \displaystyle\int_1^4 \sqrt{1+\left(\frac{1}{2\sqrt{y}}-1\right)^2}\,dy \approx 3.6095$.

15. $y = 1 + 6x^{3/2} \Rightarrow dy/dx = 9x^{1/2} \Rightarrow 1 + (dy/dx)^2 = 1 + 81x$.

So $L = \int_0^1 \sqrt{1+81x}\,dx = \int_1^{82} u^{1/2}\cdot\frac{1}{81}\,du = \frac{1}{81}\cdot\frac{2}{3}u^{3/2}\Big]_1^{82} = \frac{2}{243}\left(82\sqrt{82}-1\right)$.

21. The line at the top of the region has length $3-(-3) = 6$. Then for 21

$y = x^2 - 5$, $y' = 2x \Rightarrow 1 + (y')^2 = 1 + (2x)^2 = 1 + 4x^2$.

So the length of the curve is $L = \int_{-2}^{3} \sqrt{1+(2x)^2}\,dx = 2\int_0^3 \sqrt{1+4x^2}\,dx \overset{\text{CAS}}{\approx} 2(9.747088759) \approx 19.494$.

Thus the perimeter of the given region is $P \approx 25.494$.

31. $G(x) = \int_0^x \sqrt{t^2+6t+8}\,dt \Rightarrow G'(x) = \dfrac{d}{dx}\left[\int_0^x \sqrt{t^2+6t+8}\,dt\right] = \sqrt{x^2+6x+8}$ and

$1 + [G'(x)]^2 = 1 + x^2 + 6x + 8 = x^2 + 6x + 9 = (x+3)^2$. So the arc length for $2 \le x \le 4$ is

$L = \int_2^4 \sqrt{(x+3)^2}\,dx = \int_2^4 |x+3|\,dx = \int_2^4 (x+3)\,dx = \left[\frac{1}{2}x^2 + 3x\right]_2^4 = (8+12)-(2+6) = 12$, **(C)**.