Parts

p. 550: 9-21 odd, 29-39 odd, 41-47, 81

9. Let $u = x,\ dv = \cos 5x\,dx \Rightarrow du = dx, v = \frac{1}{5}\sin 5x.$

   Then $\int x\cos 5x\,dx = \frac{1}{5}x\sin 5x - \int \frac{1}{5}\sin 5x\,dx = \frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$

11. Let $u = t,\ dv = e^{-3t}\,dt \Rightarrow du = dt,\ v = -\frac{1}{3}e^{-3t}.$

    Then $\int te^{-3t}\,dt = -\frac{1}{3}te^{-3t} - \int -\frac{1}{3}e^{-3t}\,dt = -\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C.$

13. First let $u = x^2 + 2x,\ dv = \cos x\,dx \Rightarrow du = (2x+2)\,dx, v = \sin x.$ Then

    $I = \int (x^2 + 2x)\cos x\,dx = (x^2 + 2x)\sin x - \int (2x+2)\sin x\,dx.$ Next let $U = 2x + 2,\ dV = \sin x\,dx \Rightarrow$

    $dU = 2\,dx, V = -\cos x,$ so $\int (2x+2)\sin x\,dx = -(2x+2)\cos x - \int -2\cos x\,dx = -(2x+2)\cos x + 2\sin x.$

    Thus, $I = (x^2 + 2x)\sin x + (2x+2)\cos x - 2\sin x + C.$

15. Let $u = \cos^{-1} x,\ dv = dx \Rightarrow du = \dfrac{-1}{\sqrt{1-x^2}}dx, v = x.$ Then

    $$\int \cos^{-1} x\,dx = x\cos^{-1} x - \int \frac{-x}{\sqrt{1-x^2}}dx = x\cos^{-1} x - \int \frac{1}{\sqrt{t}}\left(\frac{1}{2}dt\right) \qquad \begin{bmatrix} t = 1 - x^2, \\ dt = -2x\,dx \end{bmatrix}$$

    $$= x\cos^{-1} x - \tfrac{1}{2}\cdot 2t^{1/2} + C = x\cos^{-1} x - \sqrt{1-x^2} + C.$$

17. Let $u = \ln t,\ dv = t^4 dt \Rightarrow du = \dfrac{1}{t}dt, v = \dfrac{1}{5}t^5.$

    Then $\int t^4 \ln t\,dt = \dfrac{1}{5}t^5 \ln t - \int \dfrac{1}{5}t^5 \cdot \dfrac{1}{t}dt = \dfrac{1}{5}t^5 \ln t - \int \dfrac{1}{5}t^4\,dt = \dfrac{1}{5}t^5 \ln t - \dfrac{1}{25}t^5 + C.$

19. Let $u = t,\ dv = \csc^2 t\,dt \Rightarrow du = dt, v = -\cot t.$ Then

    $$\int t\csc^2 t\,dt = -t\cot t - \int -\cot t\,dt = t = -t\cot t + \int \frac{\cos t}{\sin t}dt = -t\cot t + \int \frac{1}{z}dz \qquad \begin{bmatrix} z = \sin t, \\ dz = \cos t\,dt \end{bmatrix}$$

    $$= -t\cot t + \ln|z| + C = -t\cot t + \ln|\sin t| + C.$$

21. $\int \dfrac{z}{10^z}dz = \int z10^{-z}\,dz.$ Let $u = z,\ dv = 10^{-z}dz \Rightarrow du = dz, v = \dfrac{-10^{-z}}{\ln 10}.$ Then

    $$\int z10^{-z}\,dz = \frac{-z10^{-z}}{\ln 10} - \int \frac{-10^{-z}}{\ln 10}dx = \frac{-z10^{-z}}{\ln 10} - \frac{-10^{-z}}{(\ln 10)(\ln 10)} + C = \frac{z}{10^z \ln 10} + \frac{1}{10^z(\ln 10)^2} + C.$$

29. First let $u = x^2 + 1,\ dv = e^{-x}\,dx \Rightarrow du = 2x\,dx, v = -e^{-x}.$ Then

    $\int_0^1 (x^2+1)e^{-x}\,dx = \left[-(x^2+1)e^{-x}\right]_0^1 + \int_0^1 2xe^{-x}\,dx = -2e^{-1} + 1 + 2\int_0^1 xe^{-x}\,dx.$ Next let

    $U = x,\ dV = e^{-x}\,dx \Rightarrow dU = dx,\ V = -e^{-x}.$ Then

    $\int_0^1 xe^{-x}\,dx = \left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\,dx = -e^{-1} + \left[-e^{-x}\right]_0^1 = -e^{-1} - e^{-1} + 1 = -2e^{-1} + 1.$ So

    $\int_0^1 (x^2+1)e^{-x}\,dx = -2e^{-1} + 1 + 2\left(-2e^{-1} + 1\right) = -2e^{-1} + 1 - 4e^{-1} + 2 = -6e^{-1} + 3.$

31. Let $u = \ln R,\ dv = \dfrac{1}{R^2}dR \Rightarrow du = \dfrac{1}{R}dR, v = -\dfrac{1}{R}$. Then

$$\int_1^5 \frac{\ln R}{R^2}dR = \left[-\frac{1}{R}\ln R\right]_1^5 - \int_1^5 -\frac{1}{R^2}dR = -\tfrac{1}{5}\ln 5 - 0 - \left[\frac{1}{R}\right]_1^5 = -\tfrac{1}{5}\ln 5 - \left(\tfrac{1}{5}-1\right) = \tfrac{4}{5} - \tfrac{1}{5}\ln 5.$$

33. $\sin 2x = 2\sin x\cos x$, so $\int_0^\pi x\sin x\cos x\,dx = \frac{1}{2}\int_0^\pi x\sin 2x\,dx$. Let $u = x,\ dv = \sin 2x\,dx \Rightarrow$

$du = dx, v = -\frac{1}{2}\cos 2x$. Then

$$\tfrac{1}{2}\int_0^\pi x\sin 2x\,dx = \left[-\tfrac{1}{2}x\cos 2x\right]_0^\pi - \tfrac{1}{2}\int_0^\pi -\tfrac{1}{2}\cos 2x\,dx = -\tfrac{1}{2}\pi - 0 + \tfrac{1}{2}\left[\tfrac{1}{2}\sin 2x\right]_0^\pi = -\tfrac{\pi}{2}.$$

35. Let $u = M,\ dv = e^{-M}\,dM \Rightarrow du = dM, v = -e^{-M}$.

Then $\displaystyle\int_1^5 \frac{M}{e^M}dM = \int_1^5 Me^{-M}\,dM = \left[-Me^{-M}\right]_1^5 - \int_1^5 -e^{-M}\,dM = -5e^{-5} + e^{-1} - \left[e^{-M}\right]_1^5$

$$= -5e^{-5} + e^{-1} - \left(e^{-5} - e^{-1}\right) = 2e^{-1} - 6e^{-5}$$

37. Let $u = \ln(\cos x),\ dv = \sin x\,dx \Rightarrow du = \dfrac{1}{\cos x}(-\sin x)\,dx, v = -\cos x$.

Then $\int_0^{\pi/3}\sin x\ln(\cos x)\,dx = \left[-\cos x\ln(\cos x)\right]_0^{\pi/3} - \int_0^{\pi/3}\sin x\,dx = -\tfrac{1}{2}\ln\tfrac{1}{2} - 0 + \left[\cos x\right]_0^{\pi/3}$

$$= -\tfrac{1}{2}\ln\tfrac{1}{2} + \left(\tfrac{1}{2} - 1\right) = \tfrac{1}{2}\ln 2 - \tfrac{1}{2}.$$

39. Let $u = (\ln x)^2,\ dv = x^4\,dx \Rightarrow du = 2\dfrac{\ln x}{x}dx, v = \dfrac{x^5}{5}$. So,

$\displaystyle\int_1^2 x^4(\ln x)^2\,dx = \left[\frac{x^5}{5}(\ln x)^2\right]_1^2 - 2\int_1^2 \frac{x^4}{5}\ln x\,dx = \tfrac{32}{5}(\ln 2)^2 - 0 - 2\int_1^2 \frac{x^4}{5}\ln x\,dx.$ Let $U = \ln x$,

$dV = \dfrac{x^4}{5}dx \Rightarrow dU = \dfrac{1}{x}dx, V = \dfrac{x^5}{25}$. Then

$\displaystyle\int_1^2 \frac{x^4}{5}\ln x\,dx = \left[\frac{x^5}{25}\ln x\right]_1^2 - \int_1^2 \frac{x^4}{25}dx = \tfrac{32}{25}\ln 2 - 0 - \left[\frac{x^5}{125}\right]_1^2 = \tfrac{32}{25}\ln 2 - \left(\tfrac{32}{125} - \tfrac{1}{125}\right).$ So,

$$\int_1^2 x^4(\ln x)^2\,dx = \tfrac{32}{5}(\ln 2)^2 - 2\left(\tfrac{32}{25}\ln 2 - \tfrac{31}{125}\right) = \tfrac{32}{5}(\ln 2)^2 - \tfrac{64}{25}\ln 2 + \tfrac{62}{125}.$$

41. Let $u = x^2,\ dv = e^x\,dx$. Then $du = 2x\,dx\quad v = e^x$ so $\int_0^1 x^2e^x\,dx = x^2e^x\Big|_0^1 - \int_0^1 2xe^x\,dx$.

Now let $u = 2x,\ dv = e^x\,dx$ and $du = 2\,dx\quad v = e^x$ so

$$\int_0^1 x^2e^x\,dx = x^2e^x\Big|_0^1 - \left(2xe^x\Big|_0^1 - \int_0^1 2e^x\,dx\right) = \left(x^2e^x - 2xe^x + 2e^x\Big|\right)_0^1 = e - 2e + 2e - (0 - 0 + 2) = e - 2,\ \textbf{(B)}.$$

42. Let $u = x,\ dv = \sin x\,dx$. Then $du = dx\quad v = -\cos x$, so

$$\int_{\pi/2}^\pi x\sin x\,dx = -x\cos x\Big|_{\pi/2}^\pi - \int_{\pi/2}^\pi -\cos x\,dx = \left(-x\cos x + \sin x\right)\Big|_{\pi/2}^\pi$$

$$= \left(-\pi\cos\pi + \sin\pi\right) - \left(-\tfrac{\pi}{2}\cos\tfrac{\pi}{2} + \sin\tfrac{\pi}{2}\right) = (\pi - 0) - (0 + 1) = \pi - 1,\ \text{choice } \textbf{(A)}.$$

43. For $m \neq 0$, let $u = x$, $dv = e^{-x}\,dx$ and $du = dx$ $v = -e^{-x}$.

Then $\int_0^m \frac{x}{e^x}\,dx = \int_0^m xe^{-x}\,dx = -xe^{-x}\Big|_0^m + \int_0^m e^{-x}\,dx = \left(-xe^{-x} - e^{-x}\right)\Big|_0^m = \left(-\frac{m}{e^m} - \frac{1}{e^m}\right) - (-1) = \frac{e^m - m - 1}{e^m}$,
which is option (**C**).

44. Let $u = \ln x$, $dv = x\,dx$. Then $du = \frac{1}{x}dx$ $v = \frac{1}{2}x^2$, so

$$\int_1^2 x\ln x\,dx = x^2\ln x\Big|_1^2 - \int_1^2 \frac{1}{2}x^2\frac{1}{x}dx = x^2\ln x\Big|_1^2 - \int_1^2 \frac{1}{2}x\,dx = \left(x^2\ln x - \frac{x^2}{4}\right)\Bigg|_1^2$$

$= (4\ln 2 - 1) - \left(0 - \frac{1}{4}\right) = 4\ln 2 - 0.75$, choice (**A**).

45. Let $u = f(x)$, $dv = g'(x)\,dx$. Then $du = f'(x)dx$ and $v = g(x)$.

Therefore, $\int f(x)\cdot g'(x)\,dx = f(x)g(x) - \int g(x)\cdot f'(x)\,dx$, option (**D**).

46. By Exercise 45, $\int_1^3 f(x)\cdot g'(x)dx = f(x)\cdot g(x)\Big|_1^3 - \int_1^3 f'(x)\cdot g(x)dx = f(x)\cdot g(x)\Big|_1^3 - \int_1^3 h'(x)dx$

$= \left[f(x)\cdot g(x) - h(x)\right]\Big|_1^3 = \left(f(3)\cdot g(3) - h(3)\right) - \left(f(1)\cdot g(1) - h(1)\right)$

$= (5\cdot 6 - 8) - (2\cdot 4 - 7) = 21$, (**B**).

47. By Exercise 45, $\int_1^2 f(x)\cdot g'(x)dx = f(x)\cdot g(x)\Big|_1^2 - \int_1^2 f'(x)\cdot g(x)dx = f(x)\cdot g(x)\Big|_1^2 - 5$

$= \left[f(2)\cdot g(2)\right] - \left[f(1)\cdot g(1)\right] - 5 = (2\cdot 3) - \left(3\cdot(-1)\right) - 5 = 4$, (**C**).

81. For $I = \int_1^4 x\cdot f''(x)\,dx$, let $u = x$, $dv = f''(x)\,dx \Rightarrow du = dx$, $v = f'(x)$. Then

$$I = \left[x\cdot f'(x)\right]_1^4 - \int_1^4 f'(x)\,dx = 4\cdot f'(4) - 1\cdot f'(1) - \left[f(4) - f(1)\right] = 4\cdot 3 - 1\cdot 5 - (7-2) = 12 - 5 - 5 = 2$$