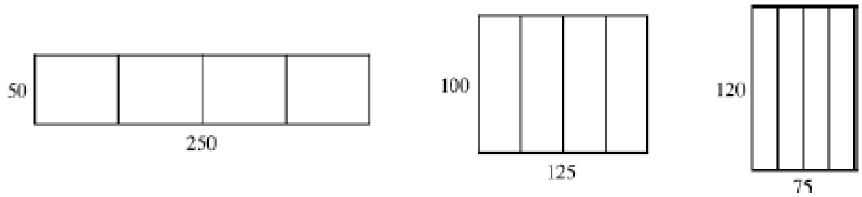
Practice Optimization

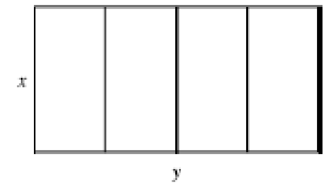
p. 374: 18, 22, 23, 27, 34, 42, 49

18. (a)



The areas of the three figures are 12,500, 12,500 and 9000 ft^2. There appears to be a maximum area of at least 12,500 ft^2.

(b) Let x denote the length of each of two sides and three dividers. Let y denote the length of the other two sides.



(c) Area $= A =$ length $\times$ width $= y \cdot x$

(d) Length of fencing $= 750 \Rightarrow 5x + 2y = 750$

(e) $5x + 2y = 750 \Rightarrow y = 375 - \frac{5}{2}x \Rightarrow A(x) = \left(375 - \frac{5}{2}x\right)x = 375x - \frac{5}{2}x^2$

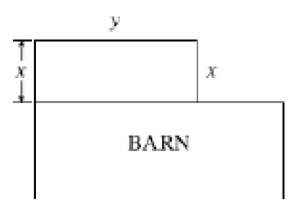
(f) $A'(x) = 375 - 5x = 0 \Rightarrow x = 75$. Since $A''(x) = -5 < 0$, there is an absolute maximum when $x = 75$. Then $y = \frac{375}{2} = 187.5$. The largest areas is $75\left(\frac{375}{2}\right) = 14{,}062.5$ ft^2. These values of x and y are between the values in the first and second figures in part (a). Our original estimate was low.

22. The volume of the crate is $V = (\text{length}) \times (\text{width}) \times (\text{height}) = l \times s \times s = ls^2 = 1200$ ft^3. Thus, $l = 200 / s^2$. Building the crate requires two square ends $(2s^2)$ and the bottom, plus two sides of length l and height s $(3sl)$. Thus the material required is

$A(s) = 2s^2 + 3sl = 2s^2 + 3s\left(1200 / s^2\right) = 2s^2 + 3600 / s$. This is option (**B**).

23. Let b be the length of the base of the box and h the height. The volume is $32{,}000 = b^2h \Rightarrow$ $h = 32{,}000 / b^2$. The surface area of the open box is $S = b^2 + 4bh = b^2 + 4(32{,}000 / b^2)b$ $= b^2 + 4(32{,}000 / b)$. So $S'(b) = 2b - 4(32{,}000) / b^2 = 2(b^3 - 64{,}000) / b^2 = 0 \Leftrightarrow b = \sqrt[3]{64{,}000} = 40$. This gives an absolute minimum since $S'(b) < 0$ if $b < 40$ and $S'(b) > 0$ if $b > 40$. The box should be $40 \times 40 \times 20$.

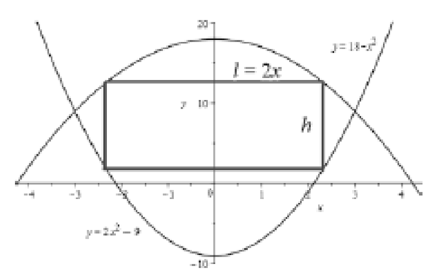
27. See the figure. The fencing costs \$20 per linear foot to install and the cost of the fencing on the west side will be split with the neighbor, so the farmer's cost C will be $C = \frac{1}{2}(20x) + 20y + 20x = 20y + 30x$. The area A will be maximized when $C = 5000$, so $5000 = 20y + 30x \Leftrightarrow$



$20y = 5000 - 3x \Leftrightarrow y = 250 - \frac{3}{2}x$. Now $A = xy = \left(250 - \frac{3}{2}x\right) = 250x - \frac{3}{2}x^2 \Rightarrow A'(x) = 250 - 3x$.

$A' = 0 \Leftrightarrow x = \frac{250}{3}$ and since $A'' = -3 < 0$, we have a maximum for A when $x = \frac{250}{3}$ ft and $y = 250 - \frac{3}{2}\left(\frac{250}{3}\right) = 125$ ft. [The maximum area is $125\left(\frac{250}{3}\right) = 10{,}416.\overline{6}$ ft^2.]

34. See the figure. The rectangle has length $l = 2x$ $(x > 0)$, and height $h = y_1 - y_2 = (18 - x^2) - (2x^2 - 9) = 27 - 3x^2$. Therefore the area of the rectangle is $A(x) = 2x \cdot (27 - 3x^2) = 54x - 6x^3$. We want to maximize the area, so we find $A'(x) = 54 - 18x^2 = 0 \Leftrightarrow 54 = 18x^2 \Leftrightarrow 3 = x^2 \Leftrightarrow$ $x = \sqrt{3}$ since $x > 0$. $A''(x) = -24x \Rightarrow A''\left(\sqrt{3}\right) = -24\sqrt{3} < 0 \Rightarrow$ the



maximum area occurs when $x = \sqrt{3}$. Then the maximum area is $A\left(\sqrt{3}\right) = 54 \cdot \sqrt{3} - 6\left(\sqrt{3}\right)^3 = 36\sqrt{3}$, which is option (**D**).

42. The volume of the cylinder is $V = 16\pi = \pi r^2 h$, which means $16 = r^2 h \Rightarrow 16h^{-1} = r^2 \Rightarrow 4h^{-1/2} = r$. The surface area (which is the amount of tin required) is $A = 2\pi rh + 2\pi r^2$, so $A(h) = 2\pi\left(4h^{-1/2}\right)h + 2\pi\left(4h^{-1/2}\right)^2 = 8\pi\sqrt{h} + 32\pi h^{-1}$. To minimize the area, we find $A'(h) = 8\pi \cdot \frac{1}{2\sqrt{h}} - \frac{32\pi}{h^2} = 0 \Leftrightarrow \frac{4\pi}{\sqrt{h}} = \frac{32\pi}{h^2} \Leftrightarrow h^2 = 8\sqrt{h} = \Leftrightarrow h^{3/2} = 8 \Leftrightarrow h = 4$. $A'(h) < 0$ for a $h < 4$, and $A'(h) > 0$ for $h > 4$, so the surface area is minimized when $h = 4$ inches. The height that will minimize the amount of tin required to construct the can is 4 (**D**).

49. $xy = 180$, so $y = 180/x$. The printed area is $A(x) = (x-2)(y-3) = (x-2)(180/x - 3) = 186 - 3x - 360/x$. $A'(x) = -3 + 360/x^2 = 0$ when $x^2 = 120 \Rightarrow x = 2\sqrt{30}$. This gives an absolute maximum since $A'(x) > 0$ for $0 < x < 2\sqrt{30}$ and $A'(x) < 0$ for $x > 2\sqrt{30}$. When $x = 2\sqrt{30}$, $y = 180/\left(2\sqrt{30}\right)$, so the dimensions are $2\sqrt{30}$ in. and $90/\sqrt{30}$ in.