Substitution

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5. Let $u = 2x$. Then $du = 2dx$ and $dx = \frac{1}{2}du$, so $\int \cos 2x\,dx = \int \cos u\left(\frac{1}{2}du\right) = \frac{1}{2}\sin u + C = \frac{1}{2}\sin 2x + C$.

9. Let $u = x^4 - 5$. Then $du = 4x^3dx$ and $x^3dx = \frac{1}{4}du$,

$$\text{so} \int \frac{x^3}{x^4-5}dx = \int \frac{1}{u}\left(\frac{1}{4}du\right) = \tfrac{1}{4}\ln|u| + C = \tfrac{1}{4}\ln|x^4 - 5| + C.$$

13. Let $u = 1-2x$. Then $du = -2dx$ and

$dx = -\frac{1}{2}du$, so $\int (1-2x)^9\,dx = \int u^9\left(-\frac{1}{2}du\right) = -\frac{1}{2}\cdot\frac{1}{10}u^{10} + C = -\frac{1}{20}(1-2x)^{10} + C$.

17. Let $u = 5-3x$. Then $du = -3\,dx$ and $dx = -\frac{1}{3}du$, so $\int \frac{dx}{5-3x} = \int \frac{du}{-3u} = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|5-3x| + C$.

21. Let $x = 1-e^u$. Then $dx = -e^u\,du$ and $e^u\,du = -dx$,

$$\text{so} \int \frac{e^u}{(1-e^u)^2}du = \int -\frac{1}{x^2}dx = -\int x^{-2}dx = -(-x^{-1}) + C = \frac{1}{x} + C = \frac{1}{1-e^u} + C.$$

25. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$ so $\int \dfrac{(\ln x)^2}{x}dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$.

29. Let $u = 1+e^x$. Then $u^2 = 1+e^x$, and $2u\,du = e^x dx$, so

$$\int e^x\sqrt{1+e^x}\,dx = \int u\cdot 2u\,du = \tfrac{2}{3}u^3 + C = \tfrac{2}{3}(1+e^x)^{3/2} + C.$$

33. Let $u = 5^t$. Then $du = 5^t\ln 5\,dt$ and $5^t\,dt = \dfrac{1}{\ln 5}du$,

$$\text{so} \int 5^t\sin\left(5^t\right)dt = \int \sin u\left(\frac{1}{\ln 5}du\right) = -\frac{1}{\ln 5}\cos u + C = -\frac{1}{\ln 5}\cos\left(5^t\right) + C.$$

37. Let $u = 1+5t$. Then $du = 5\,dt$ and $dt = \frac{1}{5}du$,

$$\text{so} \int \cos(1+5t)\,dt = \int \tfrac{1}{5}\cos u\,du = \tfrac{1}{5}\sin u + C = \tfrac{1}{5}\sin(1+5t) + C.$$

41. Let $u = \sin x$. Then $du = \cos x dx$ so $\int \sqrt{\sin x}\cos x\,dx = \int \sqrt{u}\,du = \int u^{1/2}du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\sqrt{\sin^3 x} + C$.

45. $\int \cot x\,dx = \int \dfrac{\cos x}{\sin x}dx$. Let $u = \sin x$. Then $du = \cos x\,dx$,

$$\text{so} \int \cot x\,dx = \int \frac{1}{u}du = \ln|u| + C = \ln|\sin x| + C.$$

49. Let $u = 1+x^2$. Then $du = 2x\,dx$,

$$\text{so} \int \frac{1+x}{1+x^2}dx = \int \frac{1}{1+x^2}dx + \int \frac{x}{1+x^2}dx = \tan^{-1}x + \int \frac{\frac{1}{2}du}{u} = \tan^{-1}x + \tfrac{1}{2}\ln|u| + C$$

$$= \tan^{-1}x + \tfrac{1}{2}\ln|1+x^2| + C = \tan^{-1}x + \tfrac{1}{2}\ln\left(1+x^2\right) + C \quad \left[\text{since } 1+x^2 > 0\right].$$

53. Let $u = 2x^2+1$. Then $du = 4x\,dx$ so $\int x\sqrt{2x^2+1}\,dx = \int \sqrt{2x^2+1}\,x\,dx = \int \frac{1}{4}\sqrt{u}\,du$, which is option (C).