Polar Coordinates

Rectangular (Cartesian) coordinates plot a point by moving left/right and up/down (making a rectangle)

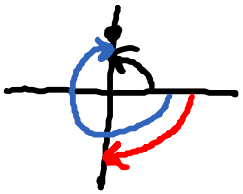
→Polar coordinates find the same point in a different way

$r =$ distance from the origin (radius)
$\theta =$ angle with positive x-axis

Ex. The Cartesian coordinates of a point are given, find the polar coordinates.

(r, θ)

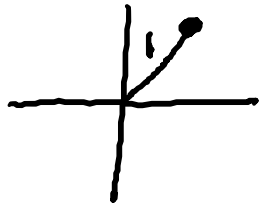
a) $(0,1)$



$\left(1, \frac{\pi}{2}\right)$ $\left(1, -\frac{3\pi}{2}\right)$

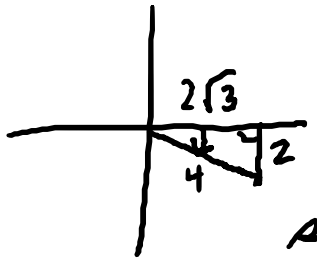
$\left(-1, -\frac{\pi}{2}\right)$

b) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



$\left(1, \frac{\pi}{4}\right)$

c) $\left(2\sqrt{3}, -2\right)$



$(2\sqrt{3})^2 + 2^2 = r^2$

$12 + 4 = r^2$

$r = 4$

$\sin\theta = \frac{2}{4}$

$\left(4, \frac{-\pi}{6}\right)$

Polar → Rect	Rect → Polar
$x = r\cos\theta$	$x^2 + y^2 = r^2$
$y = r\sin\theta$	$\tan\theta = \frac{y}{x}$

Ex. The polar coordinates of a point are given, find the Cartesian coordinates.

a) $(2, \pi)$ $x = 2\cos\pi = -2$, $y = 2\sin\pi = 0$ $(-2, 0)$

b) $\left(\sqrt{3}, \frac{\pi}{6}\right)$ $x = \sqrt{3}\cos\frac{\pi}{6} = \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2}$, $y = \sqrt{3}\sin\frac{\pi}{6} = \sqrt{3}\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

Ex. Find the polar equation for the curve.

a) $x^2 + y^2 = 3y$

$r^2 = 3r\sin\theta$

$r = 3\sin\theta$

b) $x^3y^2 + \ln y = 3$

$(r\cos\theta)^3(r\sin\theta)^2 + \ln(r\sin\theta) = 3$

Ex. Find the Cartesian equation for the curve.

a) $r = \cos\theta$

$r^2 = r\cos\theta$

$x^2 + y^2 = x$

b) $\sin\theta = r^2\cos\theta$

$r\sin\theta = r^2 r\cos\theta$

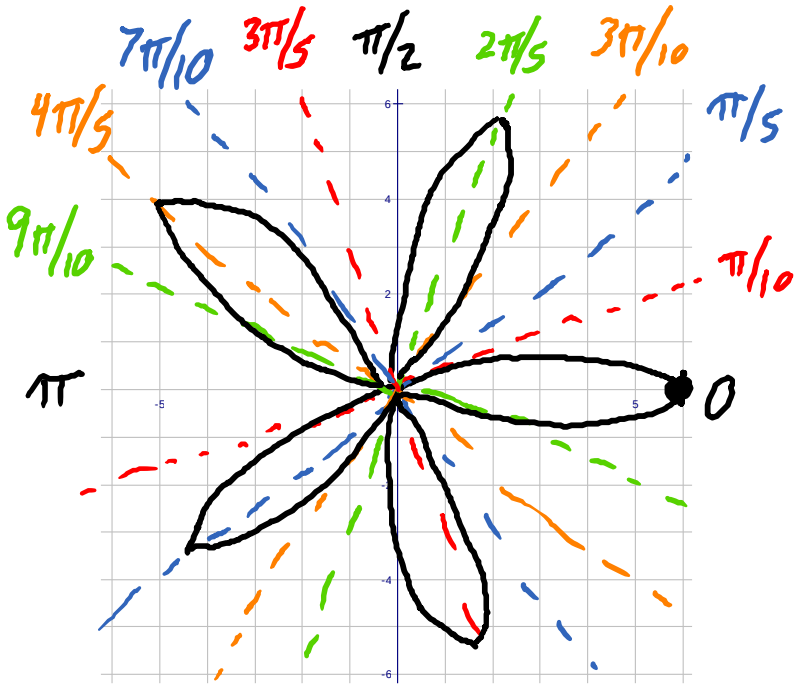
$y = (x^2 + y^2)x$

$\tan\theta = r^2$

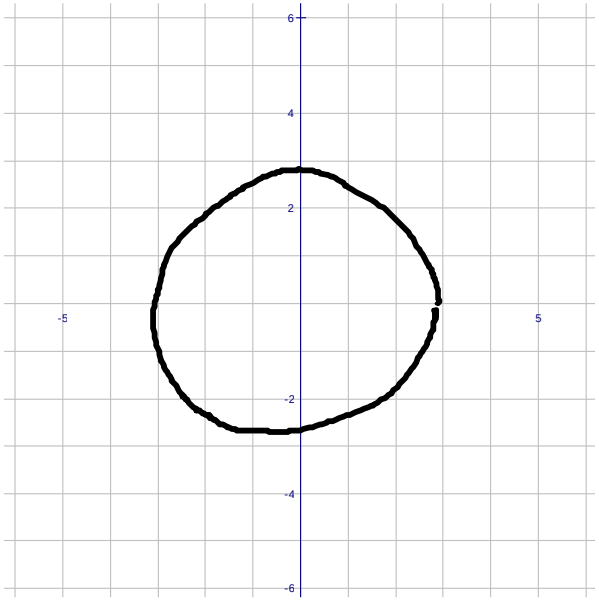
$\frac{y}{x} = x^2 + y^2$

Ex. Sketch the polar equation $r = 2\cos 5\theta$

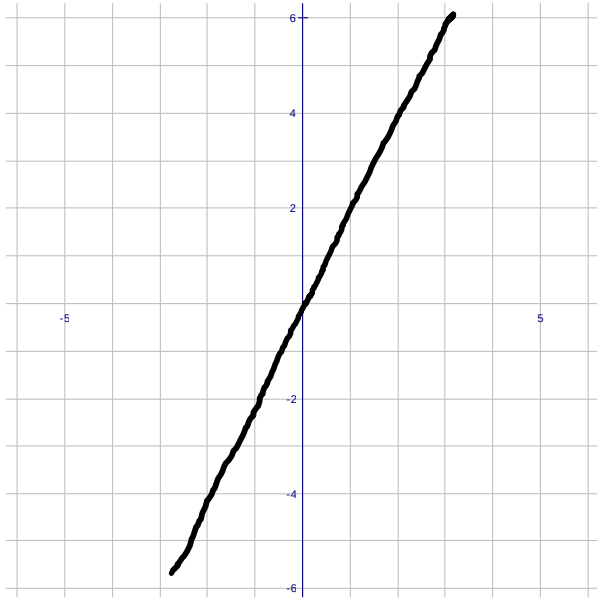
θ	$r = 2\cos 5\theta$
0	$2\cos 0 = 2$
$\pi/10$	$2\cos\frac{\pi}{2} = 0$
$\pi/5 = 2\pi/10$	$2\cos\pi = -2$
$3\pi/10$	$2\cos\frac{3\pi}{2} = 0$
$2\pi/5 = 4\pi/10$	$2\cos 2\pi = 2$
$\pi/2 = 5\pi/10$	$2\cos\frac{5\pi}{2} = 0$
$3\pi/5 = 6\pi/10$	$2\cos 3\pi = -2$
$7\pi/10$	$2\cos\frac{7\pi}{2} = 0$
$4\pi/5 = 8\pi/10$	$2\cos 4\pi = 2$
$9\pi/10$	$2\cos\frac{9\pi}{2} = 0$
$\pi = 10\pi/10$	$2\cos 5\pi = -2$



Ex. Sketch the polar equation $r = 3$



Ex. Sketch the polar equation $\theta = \frac{\pi}{3}$

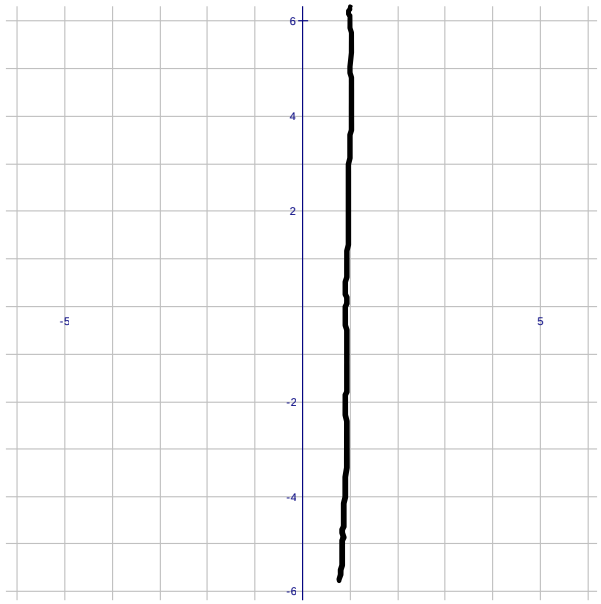


Ex. Sketch the polar equation $r = \sec\theta$.

$$r = \frac{1}{\cos\theta}$$

$$r\cos\theta = 1$$

$$x = 1$$



Ex. For the polar graph $r = 4 - \sin\theta$, find $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$

$x = r\cos\theta \Rightarrow x = (4 - \sin\theta)\cos\theta \Rightarrow \frac{dx}{d\theta} = (4 - \sin\theta)(-\sin\theta) + (\cos\theta)(-\cos\theta)$

$y = r\sin\theta \Rightarrow y = (4 - \sin\theta)\sin\theta \Rightarrow \frac{dy}{d\theta} = (4 - \sin\theta)(\cos\theta) + (\sin\theta)(-\cos\theta)$

For the function $r = f(\theta)$,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

Ex. Find the equation of the tangent line to $r = \sin\theta$ when $\theta = \frac{\pi}{6}$.

$x = r\cos\theta = \sin\theta\cos\theta$

$y = r\sin\theta = \sin^2\theta$

$\theta = \frac{\pi}{6}$

$x = \sin\frac{\pi}{6}\cos\frac{\pi}{6} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$

$y = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta(-\sin\theta) + \cos\theta(\cos\theta)}$

$m = \frac{2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{-\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$y - \frac{1}{4} = \sqrt{3}\left(x - \frac{\sqrt{3}}{4}\right)$