Warm up Problems

1. $\int (5x^3 - 4\sin x)dx$ $= \frac{5}{4}x^4 + 4\cos x + C$

2. $\int \left(\frac{1}{x} - \frac{1}{x^2}\right)dx$ $= \ln|x| + x^{-1} + C$

$\frac{1}{x^2} \to x^{-2}$

3. $\int 7e^x dx$ $= 7e^x + C$

More With Integrals

Ex. $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

Ex. $\int (2x-1)^{10} dx = \frac{1}{2} \frac{1}{11} (2x-1)^{11} + C$

Ex. $\int \frac{t^2 - 1}{t} dt = \int \frac{t^2}{t} - \frac{1}{t} dt = \int t - \frac{1}{t} dt$

$= \frac{1}{2}t^2 - \ln|t| + C$

Thm. Fundamental Theorem of Calculus

If $f(x)$ is a continuous function on $[a, b]$, and if $F'(x) = f(x)$, then

$$\boxed{\int_a^b f(x)dx = F(b) - F(a)}$$

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$$\int_a^b g'(x)dx = g(b) - g(a)$$

→The integral of the rate of change gives the total change.

$$g(b) = g(a) + \int_a^b g'(x)dx$$

→ Ending value is the starting value plus the integral of the rate.

Ex. The rate at which people enter Sea World is given by $E(t) = \frac{15600}{t^2 - 24t + 160}$. How many people entered the park during park hours, 9am to 5pm? (Assume t is hours since midnight.)

$$\int_9^{17} E(t)\,dt = 6004.270$$

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If $f(x)$ is a continuous function on $[a, b]$, and if $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

- $F(x)$ is an antiderivative of $f(x)$.

- Find an antiderivative, then plug in the endpoints

Ex. $\int_3^5 2x\,dx = x^2\Big|_3^5 = 5^2 - 3^2 = 25 - 9 = \boxed{16}$

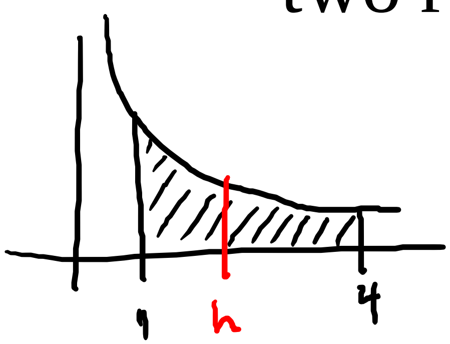
Ex. $\int_0^{\pi/2} \sin x\,dx = -\cos x\Big|_0^{\pi/2} = \left(-\cos\frac{\pi}{2}\right) - \left(-\cos 0\right) = \boxed{1}$

Ex. $\displaystyle\int_0^2 e^x dx = e^x\Big|_0^2 = e^2 - e^0 = \boxed{e^2 - 1}$

Pract. $\displaystyle\int_1^2 x^4 dx = \frac{1}{5}x^5\Big|_1^2 = \frac{32}{5} - \frac{1}{5} = \boxed{\frac{31}{5}}$

- Don't write "$+c$" on definite integrals
- We could use a calculator to get the answer, but this way we get the exact answer, not just a decimal approximation

Ex. Let R be the region bounded by $y = \frac{1}{x}$, the x-axis, $x = 1$, and $x = 4$. Find a value for h so that the line $x = h$ splits R into two regions of equal area.



$$\int_1^h \frac{1}{x}\,dx = \frac{1}{2}\int_1^4 \frac{1}{x}\,dx$$

$$\ln|x|\Big|_1^h = \frac{1}{2}\ln|x|\Big|_1^4$$

$$\ln h - \cancel{\ln 1} = \frac{1}{2}\ln 4 - \cancel{\frac{1}{2}\ln 1}$$

$$\ln h = \frac{1}{2}\ln 4$$

$$\ln h = \ln(4^{1/2})$$

$$\boxed{h = 2}$$