Rectilinear Motion

If $s(t) =$ position, then

$s'(t) = v(t) =$ velocity

$s''(t) = a(t) =$ acceleration

$|v(t)| =$ speed

$\int v(t)dt =$ displacement

$\int |v(t)|dt =$ dist. travelled

$$\begin{pmatrix}\text{ave. veloc. from} \\ t = a \text{ to } t = b\end{pmatrix} = \frac{s(b) - s(a)}{b - a}$$

$$s(b) = s(a) + \int_a^b v(t)dt$$

1) A particle moves along the y-axis such that its velocity, for $0 \le t \le 8$ is given by $v(t) = t - 3\ln(t+2) + 2t\sin\left(\frac{t^2}{12}\right)$. It is known that its initial position is $y(0) = -3$.

a) Find all values of t on the interval $4 \le t \le 7$ for which the speed is 5.

$|v(t)| = 5$

$t = 5.621, 6.520$

b) Write an expression involving an integral for $y(t)$ and use it to find the position at $t = 3$.

$y(3) = y(0) + \int_0^3 v(t)\,dt = -6.263$

$y(t) = y(0) + \int_0^t v(r)\,dr$

1) A particle moves along the y-axis such that its velocity, for $0 \leq t \leq 8$ is given by $v(t) = t - 3\ln(t+2) + 2t\sin\left(\frac{t^2}{12}\right)$. It is known that its initial position is $y(0) = -3$.

c) Find all values of t on the interval $0 \leq t \leq 8$ at which the particle changes directions. Justify your answer.

$v(t) = 0$

$t = 2.341, 6.127$

$v(t)$ changes signs at these points

d) Is the speed increasing or decreasing at $t = 5$. Justify your answer.

$v(5) = 7.877$

$a(5) = -1.772$

dec. because $v(5)$ and $a(5)$ are diff. signs

1) A particle moves along the y-axis such that its velocity, for $0 \le t \le 8$ is given by $v(t) = t - 3\ln(t+2) + 2t\sin\left(\frac{t^2}{12}\right)$. It is known that its initial position is $y(0) = -3$.

e) Find the total distance traveled by the particle on the interval $0 \le t \le 8$.

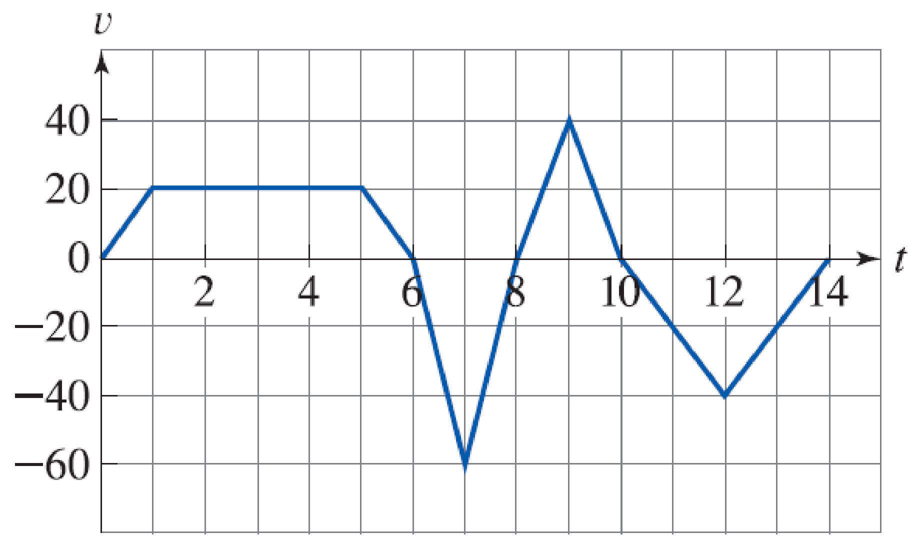
$$\int_0^8 |v(t)|\,dt = 40.287$$

2) An amusement park ride moves vertically and has velocity given in the graph below.

a) At what times t does the ride change directions? Give a reason for your answer.

$t = 6, 8, 10$

$v(t)$ changes signs



Graph of v

2) An amusement park ride moves vertically and has velocity given in the graph below.

b) If the ride starts on the ground, what is the maximum height of the ride?

$y(0) = 0$

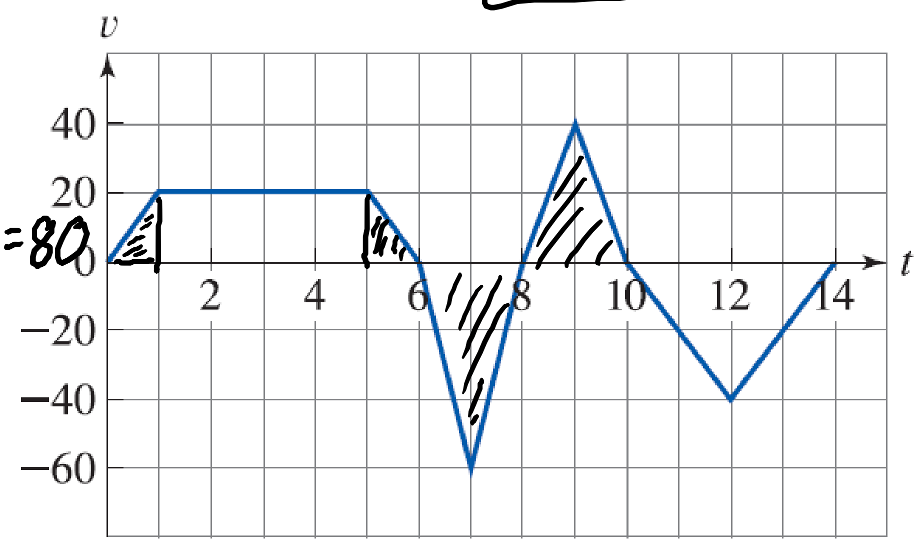
$y(6) = \int_0^6 v(t)\,dt = \frac{1}{2}(1)(20) + 4(20) + \frac{1}{2}(1)(20) = 100$

$\boxed{100}$

$\cancel{y(8)}$ = local min

$y(10) = \int_0^{10} v(t)\,dt = 100 - \frac{1}{2}(2)(60) + \frac{1}{2}(2)(40) = 80$

$\cancel{y(14)}$ = dec. up to here

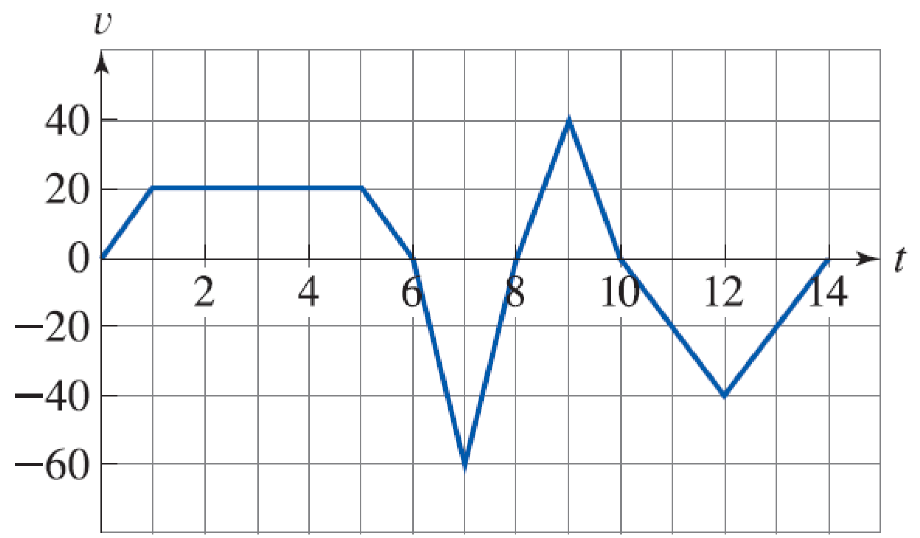


Graph of v

2) An amusement park ride moves vertically and has velocity given in the graph below.

c) Find the total distance traveled on the interval $0 \le t \le 14$.

$$\int_0^{14} |v(t)|\,dt = 100 + 60 + 40 + \frac{1}{2}(4)(40) = 280$$



Graph of v

3) A particle moves along the x-axis such that its velocity, for $0 \le t \le 10$ is given by $v(t) = t^2 - 9t + 14$. It is known that its initial position is $x(0) = 15$.

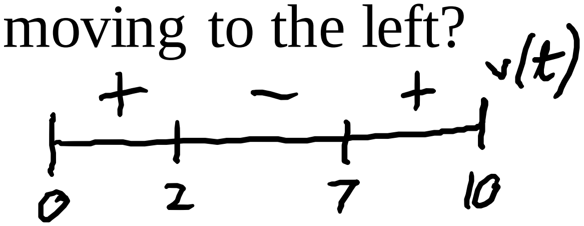
a) On what intervals is the particle moving to the left?

$v(t) = 0$

$t^2 - 9t + 14 = 0$

$(t-2)(t-7) = 0$

$t = 2, 7$



$\boxed{(2, 7)}$

b) Find the position of the particle at time $t = 8$.

$$x(8) = x(0) + \int_0^8 v(t)\,dt = 15 + \int_0^8 (t^2 - 9t + 14)\,dt = 15 + \left.\frac{1}{3}t^3 - \frac{9}{2}t^2 + 14t\right|_0^8$$

$$= 15 + \frac{8^3}{3} - \frac{9}{2}(8)^2 + 14(8)$$

4) Valerie swims in a straight line. For $0 \le t \le 50$, her velocity is a differentiable function with values given in the table below.

a) Estimate the value of $v'(30)$. $\approx \frac{v(40) - v(20)}{40-20} = \frac{1.3-(-.7)}{20} = \frac{1}{10}$

b) Using correct units, explain the meaning of $v'(30)$. $\longrightarrow \frac{m/sec}{sec.}$ sec.

At $t=30$ sec., the veloc. is changing at a rate of approx. $\frac{1}{10}$ m/sec^2.

t (sec.)	0	8	20	40	50
$v(t)$ (m/sec.)	0	1.2	-0.7	1.3	1

4) Valerie swims in a straight line. For $0 \le t \le 50$, her velocity is a differentiable function with values given in the table below.

c) Is there a point on $8 \le t \le 40$ such that $v'(t) = \frac{1}{320}$? Justify your answer.

$\frac{v(40)-v(8)}{40-8} = \frac{1.3-1.2}{32} = \frac{.1}{32} = \frac{1}{320}$ v is cont. and diff., so MVT applies

yes

d) Is there a point on $0 \le t \le 8$ such that $v(t) = \frac{1}{2}$? Justify your answer.

$v(0) < \frac{1}{2}$ diff. implies cont.

$v(8) > \frac{1}{2}$ yes, $v(t) = \frac{1}{2}$ on interval by IVT

t (sec.)	0	8	20	40	50
$v(t)$ (m/sec.)	0	1.2	-0.7	1.3	1

4) Valerie swims in a straight line. For $0 \le t \le 50$, her velocity is a differentiable function with values given in the table below.

e) Using correct units, explain the meaning of $\int_0^{50} |v(t)|\, dt$.

dist. trav. in m from $t=0$ sec. to $t=50$ sec.

f) Approximate $\int_0^{50} |v(t)|\, dt$ using a right Riemann sum with the intervals indicated in the table.

$$8\,v(8) + 12\,|v(20)| + 20 \cdot v(40) + 10\,v(50) = 54$$

t (sec.)	0	8	20	40	50
$v(t)$ (m/sec.)	0	1.2	-0.7	1.3	1

Unit 6 Progress Check: MCQ Part B

- Do #2, 8-9

Unit 6 Progress Check: MCQ Part C

- Do #1-3

Unit 8 Progress Check: MCQ Part A

- Do #8-10