New seats today…

You may sit where you wish

Trigonometric Integration

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$1 + \cot^2 x = \csc^2 x$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + c$$

We will use trigonometric identities to make integrals into substitution problems.

Ex. $\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \quad \sin x \, dx$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x \, dx \\ -du &= \sin x \, dx \end{aligned}}$$

$= \int (1 - \cos^2 x) \cos^2 x \sin x \, dx$

$= \int (1 - u^2) u^2 (-1) \, du$

$= \int (-u^2 + u^4) \, du$

$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C$

$= \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$

$\underline{\text{Ex.}}$ $\int \sin^7 x \cos^5 x \, dx = \int \sin^7 x \cos^4 x \cos x \, dx$

$= \int \sin^7 x \left(\cos^2 x\right)^2 \cos x \, dx$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \end{aligned}}$$

$= \int \sin^7 x \left(1 - \sin^2 x\right)^2 \cos x \, dx$

$= \int u^7 \left(1 - u^2\right)^2 du = \int u^7 \left(1 - 2u^2 + u^4\right) du$

$= \int u^7 - 2u^9 + u^{11} \, du = \frac{1}{8} u^8 - \frac{1}{5} u^{10} + \frac{1}{12} u^{12} + C$

$= \frac{1}{8} \sin^8 x - \frac{1}{5} \sin^{10} x + \frac{1}{12} \sin^{12} x + C$

<u>Ex.</u> $\int \sin^2 x \cos^2 x \, dx = \int \frac{1}{2}(1-\cos 2x)\cdot\frac{1}{2}(1+\cos 2x)\,dx$

$= \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int \sin^2 2x \, dx$

$= \frac{1}{4}\int \frac{1}{2}(1-\cos 4x)\,dx = \frac{1}{8}\left(x - \frac{1}{4}\sin(4x)\right) + C$

Ex. $\int \frac{\cos^3 x}{\sqrt{\sin x}} dx = \int \frac{\cos^2 x}{\sqrt{\sin x}} \cos x \, dx$

$$= \int \frac{1-\sin^2 x}{\sqrt{\sin x}} \cos x \, dx \qquad \boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$$

$$= \int \frac{1-u^2}{\sqrt{u}} du = \int \frac{1}{\sqrt{u}} - \frac{u^2}{\sqrt{u}} du = \int \left(u^{-1/2} - u^{3/2}\right) du$$

$$= 2u^{1/2} - \frac{2}{5} u^{5/2} + C$$

$$= 2(\sin x)^{1/2} - \frac{2}{5}(\sin x)^{5/2} + C$$

There is a guide for $\sin x$ and $\cos x$ on p. 555-556

$\underline{\text{Ex.}}$ $\int \sec^4 x \tan^3 x \, dx = \int \sec^2 x \tan^3 x \quad \sec^2 x \, dx$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x \, dx \end{aligned}}$$

$= \int (1+\tan^2 x) \tan^3 x \sec^2 x \, dx$

$= \int (1+u^2) u^3 \, du$

$= \int (u^3 + u^5) \, du = \frac{1}{4} u^4 + \frac{1}{6} u^6 + C$

$= \frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$

$$\underline{\text{Ex.}}\ \int \frac{\sec x}{\tan^2 x}\,dx = \int \frac{1/\cos x}{\sin^2 x/\cos^2 x}\,dx = \int \frac{1}{\cos x}\cdot\frac{\cos^2 x}{\sin^2 x}\,dx$$

$$= \int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\cos x\,dx$$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \end{aligned}}$$

$$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} + C$$

$$= \frac{-1}{\sin x} + C = -\csc x + C$$

There is a guide for $\tan x$ and $\sec x$ on p. 557

Ex. $\int x \cos^2 x \, dx$

$$\boxed{\begin{array}{ll} u = x & dv = \cos^2 x \, dx \\ du = dx & v = \frac{1}{2}x + \frac{1}{4}\sin 2x \end{array}}$$

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$$= x\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right) - \int\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right)dx$$

$$= x\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right) - \left(\frac{1}{4}x^2 - \frac{1}{8}\cos 2x\right) + C$$

$$\int \cos^2 x \, dx = \int \frac{1}{2}(1 + \cos 2x)\,dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) = \frac{1}{2}x + \frac{1}{4}\sin 2x$$

Pract.

1. $\int \cos^5 x\, dx$ $\quad$ $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$

2. $\int \sec^4 x \tan^4 x\, dx$ $\quad$ $\frac{1}{5}\tan^5 x + \frac{1}{7}\tan^7 x + c$