A Graphical Approach

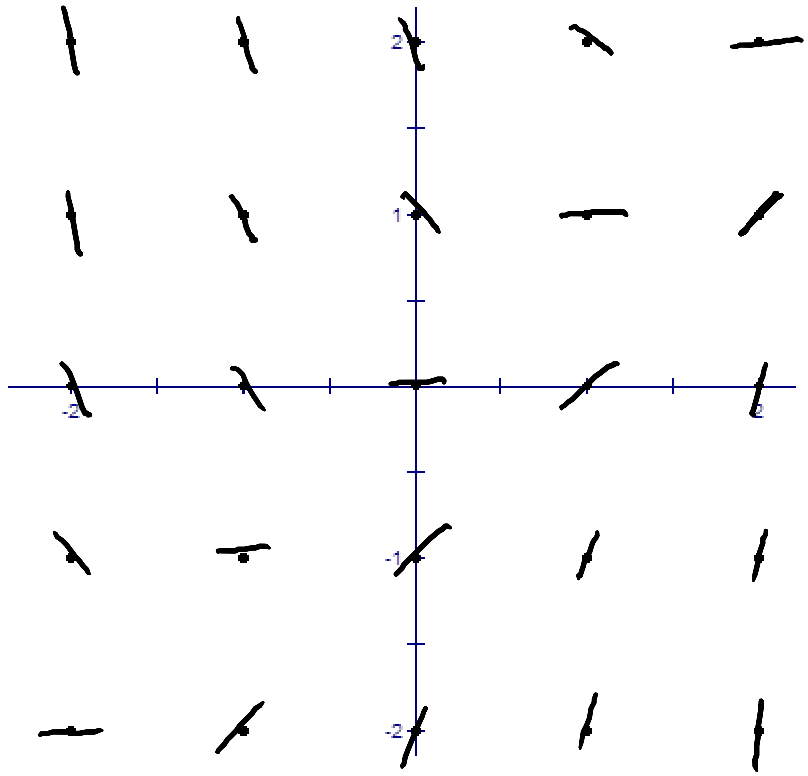
Ex. $\frac{dy}{dx} = x^2 - y$

We can't solve this differential equation, but we can find the slope of the solution at (0,2) -- assuming it passes through this point.

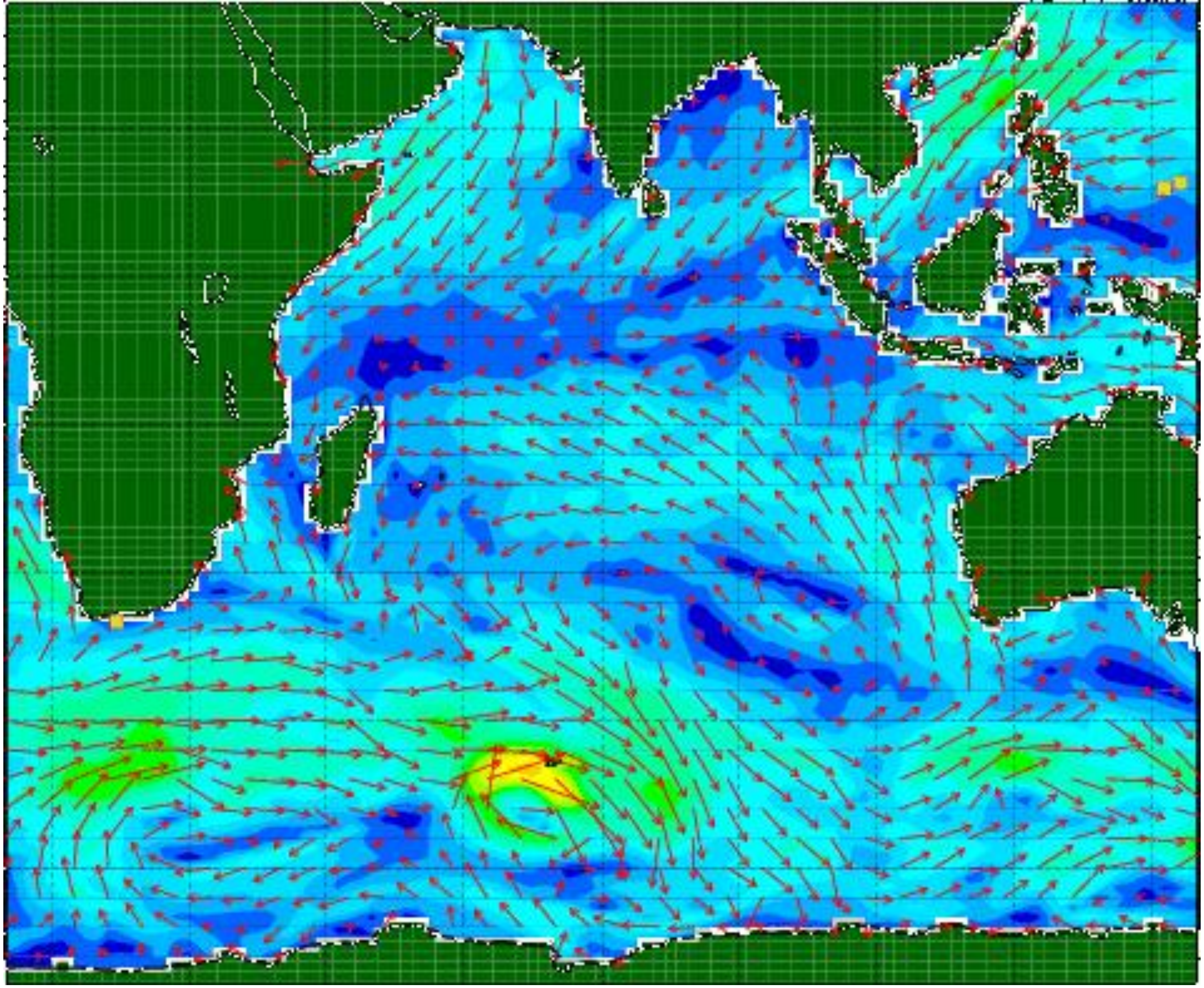
We can draw a segment through the point that has the appropriate slope: called a lineal element.

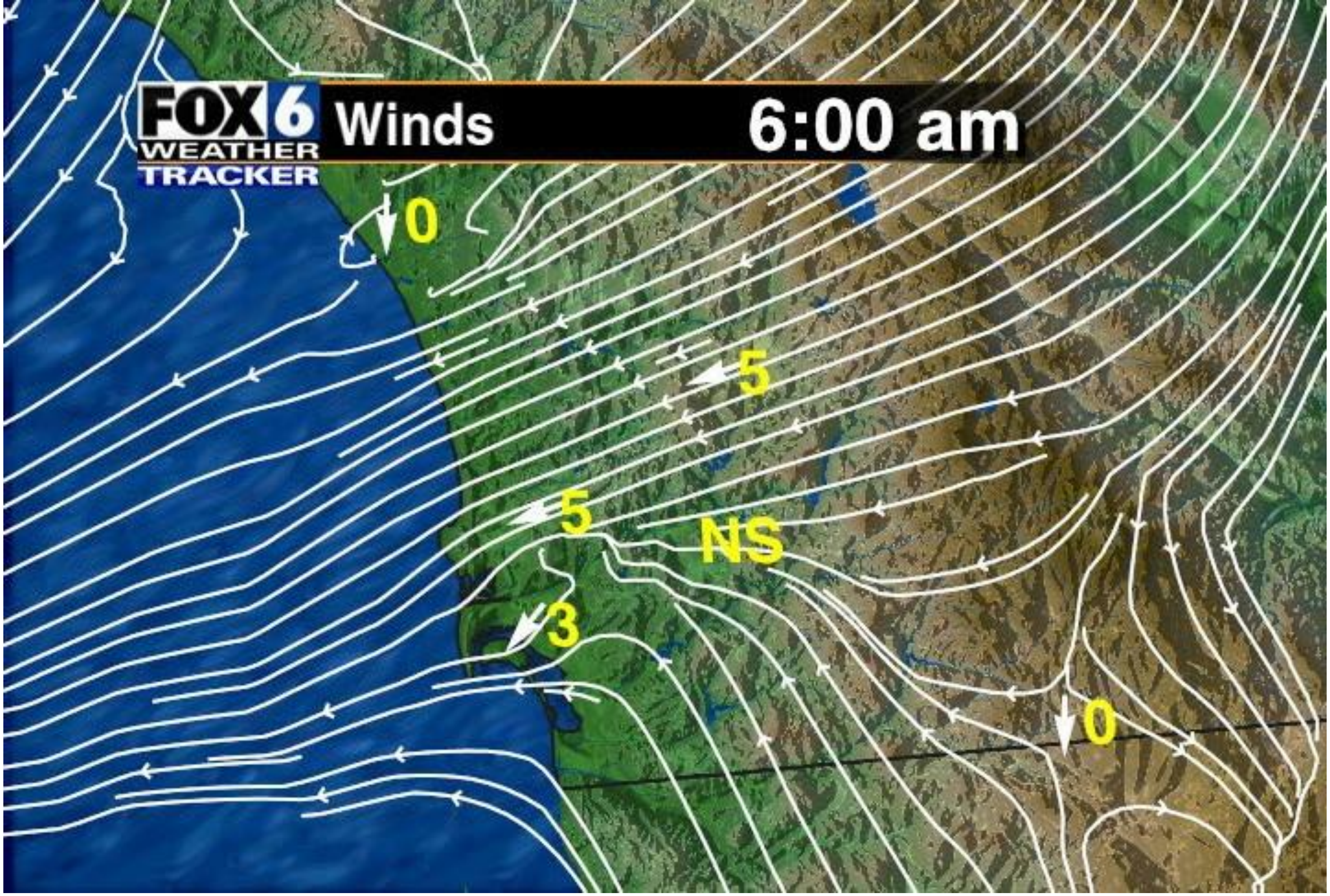
If we draw several of these lines, we get a good idea of what a solution would look like. This is called a slope field or direction field.

Ex. Draw a slope field for $\frac{dy}{dx} = x - y$, then sketch a solution that satisfies $y(0) = 0$.



Here’s what it would look like if we used lots of points…

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A Numerical Approach

The slope field gives us an idea of what the solution curve looked like.

→Euler's method will let us approximate values of the solution.

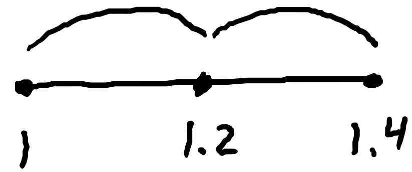
Euler's Method

Starting at the initial value, find the equation of the tangent to the solution at that point.

Follow the tangent line from the initial point for a short interval (Δx).

The point at which you end up is your new starting point, and you begin the process over.

Ex. Consider the differential equation $\frac{dy}{dx} = 3xy$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 1$. Use Euler's Method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$.



$x_1 = 1 \qquad y_1 = 1 \qquad m_1 = 3(1)(1) = 3$

$y = 1 + 3(x-1)$

$x_2 = 1.2 \qquad y_2 = 1+3(1.2-1) \qquad m_2 = 3(1.2)(1.6)$

$= 1+3(.2) = 1.6 \qquad = 5.76$

$y = 1.6 + 5.76(x-1.2)$

$\boxed{f(1.4) \approx 2.752}$

$x_3 = 1.4 \qquad y_2 = 1.6 + 5.76(1.4-1.2)$

$= 1.6 + 576(.2) = 2.752$

Ex. Redo the previous problem, using four steps of equal size.

$$\frac{dy}{dx} = 3xy$$

from DE

$y_2 = y_1 + y' \cdot \Delta x$

Δx	x_1	y_1	y'	y_2
.1	1	1	$3(1)(1)=3$	$1+3(.1)=1.3$
.1	1.1	1.3	$3(1.1)(1.3)=4.29$	$1.3+4.29(.1)=1.729$
.1	1.2	1.729	$3(1.2)(1.729)=6.224$	$1.729+6.224(.1)=2.351$
.1	1.3	2.351	$3(1.3)(2.351)=9.171$	$2.351+9.171(.1)=3.269$
	1.4	3.269		

Be sure to write out $f(1.4) \approx \ldots$

$f(1.4) \approx 3.269$

→ The table is not the end of your answer.