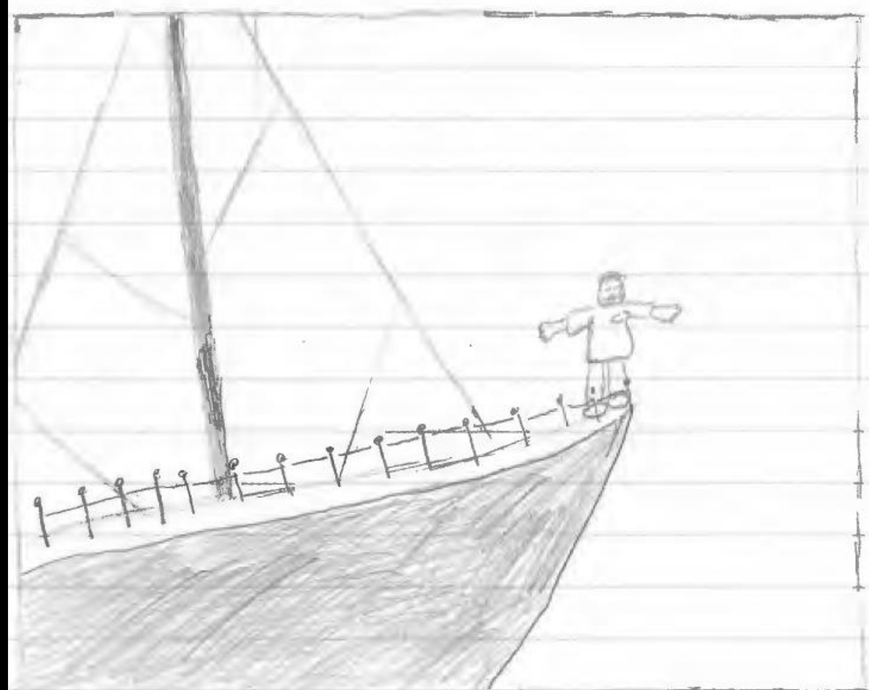
The Ti$\left(\frac{\sin}{\cos}\right)$ic



More Power Series

We can take the derivative and integral of the power series term by term

→ The radius of convergence won't change, though the endpoints of the interval might

Ex. For the function $f(x) = \sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$, find $f'(x)$.

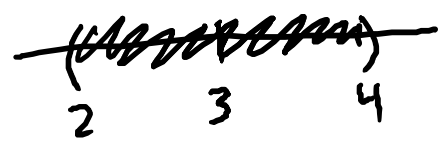
What is its interval of convergence?

$$f'(x) = \sum_{n=1}^{\infty} (x-3)^{n-1}$$

geom. series

$r = x-3$

conv. for $|x-3| < 1$



$$\boxed{(2,4)}$$

Over the next two lessons, we’ll be given a function and want to find a power series representation.

- This means a power series whose values are the same as the function
- As we go along, we’ll also address the question of why it’s helpful to find a power series representation

Recall the geometric series:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

We can use this to write some function as a power series.

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

So $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ as long as $|x| < 1$.

<u>Ex.</u> $f(x) = \dfrac{1}{1+x}$

$$= \frac{1}{1-(-x)}$$

$$= \sum_{n=0}^{\infty} (-x)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n x^n$$

Ex. $f(x) = \dfrac{1}{1-x^2}$

$$= \sum_{n=0}^{\infty} (x^2)^n$$

$$= \sum_{n=0}^{\infty} x^{2n}$$

Ex. $f(x) = \dfrac{x^3}{1-x}$

$$= x^3 \frac{1}{1-x}$$

$$= x^3 \sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=0}^{\infty} x^3 x^n$$

$$= \sum_{n=0}^{\infty} x^{n+3}$$

Ex. $f(x) = \dfrac{1}{5 - x}$

$$= \frac{1}{5\left(1 - \frac{x}{5}\right)} = \frac{1}{5} \cdot \frac{1}{1 - \frac{x}{5}}$$

$$\left|\frac{x}{5}\right| < 1$$

$$= \frac{1}{5} \sum_{n=0}^{\infty} \left(\frac{x}{5}\right)^n = \sum_{n=0}^{\infty} \frac{1}{5} \frac{x^n}{5^n}$$

$$|x| < 5$$

Show the first 4 non-zero terms and the general term. Find the interval of convergence.

$$(-5, 5)$$

$$= \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} x^n = \frac{1}{5} + \frac{1}{25}x + \frac{1}{125}x^2 + \frac{1}{625}x^3 + \dots$$

$\underline{\text{Ex.}}$ $f(x) = \tan^{-1} x$

$$= \int \frac{1}{1+x^2}\,dx = \int \frac{1}{1-(-x^2)}\,dx$$

$$= \int \sum_{n=0}^{\infty} (-x^2)^n\,dx = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

Ex. $f(x) = \ln x$

$|1-x| < 1$

$(0, 2)$

$$= \int \frac{1}{x}\,dx = \int \frac{1}{1-(1-x)}\,dx$$

$$= \int \sum_{n=0}^{\infty} (1-x)^n\,dx = \int \sum_{n=0}^{\infty} (-1)^n (x-1)^n\,dx$$

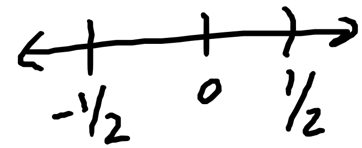
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (x-1)^{n+1}$$

Ex. Find a function for the power series and give the interval of convergence.

$$\sum_{n=0}^{\infty}(2x)^n = \frac{1}{1-2x}$$

$$|2x| < 1$$

$$|x| < \frac{1}{2}$$

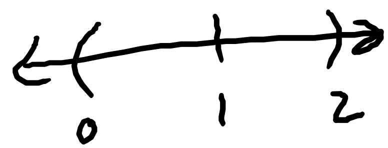


$$\boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$$

Ex. Find a function for the power series and give the interval of convergence.

$$\sum_{n=0}^{\infty} 4(x-1)^n = \frac{4}{1-(x-1)} = \frac{4}{2-x}$$

$$|x-1|<1$$



$$\boxed{(0,2)}$$

Ex. Find a function for the power series ~~and give the interval of convergence.~~

$$\sum_{n=1}^{\infty}(x^2)^n = \frac{x^2}{1-x^2}$$