Warm up Problems

1. $$\int_1^2 \frac{t}{1+3t^2}\,dt$$

2. $$\int \frac{\sin\sqrt{y}}{\sqrt{y}}\,dy$$

3. $$\int \tan x\,dx$$

Integration by Parts

To find our formula, we use the product rule (don't write this down):

$$\int d(u \cdot v) = \int u\,dv + \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

- We pick u and dv
- After using formula, you still have x's

Ex. $\int x \cos x \, dx = x \sin x - \int \sin x \, dx$

$$\int u \, dv = uv - \int v \, du$$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$$= x \sin x - (-\cos x) + C$$

When picking u, go in LIATE order:

Logarithm

Inverse Trigonometric

Algebraic (Polynomial) $\leftarrow x$

Trigonometric $\leftarrow \cos x$

Exponent

Ex. $\int x \sec^2 x \, dx$

$$\boxed{\begin{array}{ll} u = x & dv = \sec^2 x \, dx \\ du = dx & v = \tan x \end{array}}$$

L
I
A ← x
T ← $\sec^2 x$
E

$= x \tan x - \int \tan x \, dx$

$= \boxed{x \tan x - \ln|\sec x| + C}$

$$\boxed{uv - \int v \, du}$$

Ex. $\int x^2 e^x dx$

$$\boxed{\begin{array}{ll} u = x^2 & dv = e^x dx \\ du = 2x\,dx & v = e^x \end{array}}$$

$= x^2 e^x - \int e^x \cdot 2x\,dx$

$$\boxed{\begin{array}{ll} u = 2x & dv = e^x dx \\ du = 2\,dx & v = e^x \end{array}}$$

$= x^2 e^x - \left[2xe^x - \int e^x \cdot 2\,dx\right]$

$= x^2 e^x - 2xe^x + 2e^x + C$

L
I
A ←
T
E ←

1. $\int xe^{-x}dx$

$-xe^{-x} - e^{-x} + c$

2. $\int x \ln x \, dx$

$\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$

3. $\int x^2 \sin x \, dx$

$-x^2 \cos x + 2x \sin x + 2 \cos x + c$

4. $\int x \cos(3x) \, dx$

$\frac{1}{3}x \sin(3x) + \frac{1}{9}\cos(3x) + c$

Ex. $\int \ln x \, dx$

$$\boxed{\begin{array}{ll} u = \ln x & dv = 1\,dx \\ du = \frac{1}{x}dx & v = x \end{array}}$$

$$= x\ln x - \int x \cdot \frac{1}{x}dx$$

$$= x\ln x - \int 1\,dx$$

$$\boxed{\int \ln x\,dx = x\ln x - x + C}$$