Warm up Problems

1. $\int x^2(2x^3 - 5)^5 dx$

2. $\int e^{-5x} dx$

3. $\int \frac{\sqrt{\ln x}}{x} dx$

More Substitution

<u>Ex.</u> $\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx = \int \frac{1}{\cos x} \sin x\, dx$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\, dx \\ -du &= \sin x\, dx \end{aligned}}$$

$= \int \frac{1}{u}(-1)\, du = -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln|\cos x|^{-1} + C$

$$\boxed{\int \tan x\, dx = \ln|\sec x| + C}$$

Ex. $\int \frac{x}{x+3}\,dx = \int \frac{u-3}{u}\,du$

$u = x+3$

$du = dx$

$u - 3 = x$

$$= \int \frac{u}{u} - \frac{3}{u}\,du$$

$$= \int \left(1 - \frac{3}{u}\right) du$$

$$= u - 3\ln|u| + C$$

$$= x + 3 - 3\ln|x+3| + C$$

$$= x - 3\ln|x+3| + C$$

In a definite integral, you should find the antiderivative using substitution, change back to x, <u>and then</u> plug in endpoints.

<u>Ex.</u> $\displaystyle\int_0^2 x e^{x^2}\,dx$

$$\boxed{\begin{aligned} u &= x^2 \\ du &= 2x\,dx \\ \tfrac{1}{2}du &= x\,dx \end{aligned}}$$

** $= \displaystyle\int e^u \cdot \tfrac{1}{2}\,du$

* $= \tfrac{1}{2}e^u \Big|_0^4 = \tfrac{1}{2}e^{x^2}\Big|_0^2$

$$\boxed{= \tfrac{1}{2}e^4 - \tfrac{1}{2}e^0}$$

$= \displaystyle\int_0^2 e^u \tfrac{1}{2}\,du$

$= \tfrac{1}{2}e^u\Big|_0^2$

$= \tfrac{1}{2}e^2 - \tfrac{1}{2}e^0$

$$\underline{\text{Ex.}} \int_0^{\pi/4} \tan^3\theta \sec^2\theta \, d\theta$$

$$u = \tan\theta$$
$$du = \sec^2\theta \, d\theta$$

$$= \int_{*}^{**} u^3 \, du$$

$$= \frac{1}{4}u^4 \Big|_{*}^{**}$$

$$= \frac{1}{4}\tan^4\theta \Big|_0^{\pi/4}$$

$$= \frac{1}{4}\left(\tan\frac{\pi}{4}\right)^4 - \frac{1}{4}(\tan 0)^4 = \boxed{\frac{1}{4}}$$

$\theta = \pi/4 \rightarrow u = 1$

$\theta = 0 \rightarrow u = 0$

$$= \int_0^1 u^3 \, du$$

$$= \frac{1}{4}u^4 \Big|_0^1$$

$$= \frac{1}{4} - 0 = \boxed{\frac{1}{4}}$$

Pract. $\displaystyle\int_1^3 \frac{1}{5-x}\,dx$

$$\boxed{\begin{aligned} u &= 5-x \\ du &= -dx \\ -du &= dx \end{aligned}}$$

$x=3 \longrightarrow u=2$

$x=1 \longrightarrow u=4$

$$= \int_{*}^{**} \frac{1}{u}(-1)\,du$$

$$= -\ln|u|\Big|_{*}^{**}$$

$$= -\ln|5-x|\Big|_1^3$$

$$= -\ln(2) + \ln(4)$$

$$= \int_4^2 \frac{1}{u}(-1)\,du$$

$$= -\ln|u|\Big|_4^2$$

$$= -\ln 2 - (-\ln 4)$$

We could have changed the endpoints to u…