- Blue part is out of 38
- Green part is out of 62

→Total of 100 points possible

Substitution

Right now, we can only integrate a small number of functions…in this chapter we will learn two methods that will expand that list.

Ex. If $f(x) = (x^{10} - 4)^4$, find $f'(x)$.

$$f'(x) = 4(x^{10}-4)^3 \cdot 10x^9$$
$$= 40x^9(x^{10}-4)^3$$

Ex. $\int \sin(12x)\,dx = \int \sin u \cdot \frac{1}{12}\,du$

$$\boxed{\begin{aligned} u &= 12x \\ du &= 12\,dx \\ \tfrac{1}{12}\,du &= dx \end{aligned}}$$

$= \frac{-1}{12}\cos u + C$

$= -\frac{1}{12}\cos(12x) + C$

Ex. $\int x(x^2-5)^4\,dx = \int u^4 \cdot \frac{1}{2}\,du = \frac{1}{10}u^5 + C$

$$\boxed{\begin{aligned} u &= x^2-5 \\ du &= 2x\,dx \\ \tfrac{1}{2}\,du &= x\,dx \end{aligned}}$$

$= \frac{1}{10}(x^2-5)^5 + C$

Substitution is a result of the chain rule

- u will be the stuff on the inside
- its derivative will always show up on the outside

→All x's must change to u's before we can integrate

Ex. $\int \sqrt{\cos x}\,\sin x\,dx$ $= \int u^{1/2}(-1)\,du$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\,dx \\ -du &= \sin x\,dx \end{aligned}}$$

$= -\frac{2}{3}u^{3/2} + C$

$= \frac{-2}{3}(\cos x)^{3/2} + C$

Ex. $\int \frac{(\ln x)^4}{x}\,dx$ $= \int u^4\,du = \frac{1}{5}u^5 + C$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$= \frac{1}{5}(\ln x)^5 + C$

Ex. $\int \tan^5 x \sec^2 x \, dx = \int (\tan x)^5 \sec^2 x \, dx$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x \, dx \end{aligned}}$$

$= \int u^5 \, du = \frac{1}{6} u^6 + C$

$= \frac{1}{6} \tan^6 x + C$

Ex. $\int (x^3 - 2)^2 dx = \int (x^6 - 4x^3 + 4) \, dx$

$= \frac{1}{7} x^7 - x^4 + 4x + C$

1. $\displaystyle\int \frac{1}{(1+2x)^3}\,dx$

$$-\frac{1}{4}(1+2x)^{-2}+c$$

2. $\displaystyle\int \sec^2(3x)\,dx$

$$\frac{1}{3}\tan(3x)+c$$

3. $\displaystyle\int x^2\sqrt{x^3+1}\,dx$

$$\frac{2}{9}(x^3+1)^{3/2}+c$$

4. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$$-2\cos\sqrt{x}+c$$