Convergence Test Practice

1. $\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

2. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

3. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

4. $\sum_{n=1}^{\infty} \frac{5^n}{4^n+3}$

5. $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

6. $\sum_{n=1}^{\infty} ne^{-n^2}$

7. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3-2n^2+5}$

8. $\sum_{n=1}^{\infty} (-1)^n \, 2^{1/n}$

9. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}} \right)$

Test	Series	Conditions for Convergence	Conditions for Divergence	Comment
Test for Divergence	$\sum a_n$		$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to prove convergence
Geometric Series	$\sum a \cdot r^n$	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	$\text{Sum} = \frac{\text{first term}}{1-r}$
Integral Test (f continuous, positive, decreasing)	$\sum a_n$ $a_n = f(n)$	$\int_1^\infty f(x)\,dx$ converges	$\int_1^\infty f(x)\,dx$ diverges	
p-series	$\sum \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Limit Comparison Test ($a_n, b_n > 0$)	$\sum a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n}$ positive and finite $\sum b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n}$ positive and finite $\sum b_n$ diverges	
Direct Comparison Test ($a_n, b_n > 0$)	$\sum a_n$	$a_n < b_n$ and $\sum b_n$ converges	$a_n > b_n$ and $\sum b_n$ diverges	
Alternating Series Test for Convergence	$\sum (-1)^n a_n$	$\lim_{n\to\infty} a_n = 0$ and a_n decreasing		Remainder $\leq a_{N+1}$
Absolute Convergence Test	$\sum (-1)^n a_n$	$\sum a_n$ converges		
Ratio Test	$\sum a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Test fails if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Root Test	$\sum a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} > 1$	Test fails if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = 1$