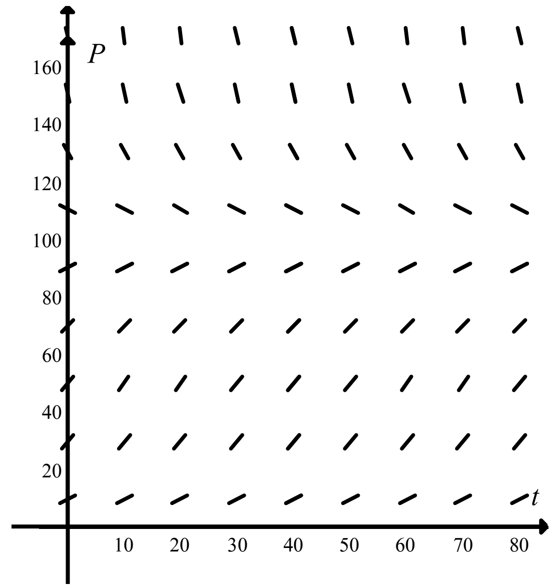
<u>Logistic Worksheet</u>

A. In 2025, right after the onset of the zombie apocalypse, there were 15 reported cases of zombiism in San Diego. Two years later, there were 35 cases. The growth rate of the zombie population z is $\frac{dz}{dt} = kz(1 - \frac{z}{5000})$, where t is years since 2025.
(a) Write a model for the zombie population in terms of t.
(b) Use the model to estimate the zombie population after 30 years.
(c) Graph the solution curve.
(d) Find the limit of the model as $t \to \infty$.
(e) Find the zombie population at the point where the population is increasing most rapidly.

B. Jellystone Park is capable of supporting no more than 100 bears. This can be modeled by a logistic differential equation with $k = 0.1$.
(a) Write the differential equation.
(b) The slope field for this differential equation is shown.
Where does there appear to be a horizontal asymptote?
What happens if the starting point is above this asymptote? Below?
(c) If the park begins with 10 bears, sketch a graph of $P(t)$ on the slope field.
(d) Write the solution to the DE with initial condition $P(0) = 10$.
(e) Find $\lim_{t\to\infty} P(t)$
(f) When will the bear population reach 50?
(g) When is the bear population growing the fastest?



1. A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{2}\left(3 - \frac{P}{20}\right)$, where the initial population $P(0) = 5$ and t is the time in years.
(a) What is $\lim_{t\to\infty} P(t)$?
(b) For what values of P is the population growing the fastest?
(c) Find the slope of the graph of P at the point of inflection.

2. A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the environment can support no more than 4000 elk. The growth rate of the elk population is $\frac{dP}{dt} = kP\left(1 - \frac{P}{4000}\right)$, $40 \le t \le 4000$. Find a model for the elk population in terms of t, and use it to predict the population after 15 years.

3. A population of rabbits is given by the formula $P(t) = \frac{1000}{1+e^{4.8-0.7t}}$, where t is the number of months after a few rabbits are released.
(a) Identify k and the carrying capacity.
(b) Estimate $P(0)$. Explain its meaning in the context of the problem.

4. The number of students infected by measles in a certain school is given by the formula $P(t) = \frac{200}{1+e^{5.3-t}}$, where t is the number of days after students are first exposed to an infected student.
(a) Identify k and the carrying capacity.
(b) Estimate $P(0)$. Explain its meaning in the context of the problem.

5. A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is $\frac{dP}{dt} = 0.0015P(150 - P)$, where time t is in weeks.
(a) Find a formula for the guppy population in terms of t.
(b) How long will it take for the guppy population to be 100? 125?

6. A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were known to be in the preserve in 1970. Assume that the rate of growth of the population is $\frac{dP}{dt} = 0.0004P(250 - P)$, where time t is in years.
(a) Find a formula for the gorilla population in terms of t.
(b) How long will it take for the gorilla population to reach the point when the population grows fastest?

7. Suppose a population develops according to the logistic equation: $\frac{dP}{dt} = 0.05P - 0.0005P^2$ where t is measured in weeks. What is the carrying capacity?

8. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?
(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

9. Which of the following differential equations for a Population P could model the logistic growth shown in the figure at right?
(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$
(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$
(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$
(D) $\frac{dP}{dt} = 0.2P^2 - 0.001P$