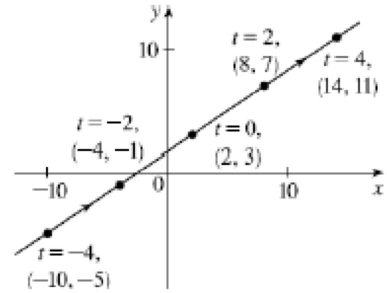
p. 765: 15-25 odd, 27-29, 44, 54-55, 65-66

15. (a) $x = 3t + 2, \quad y = 2t + 3$

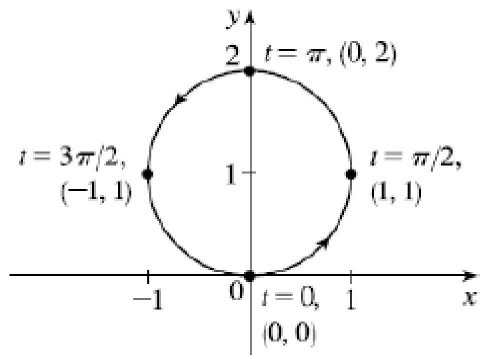


t	-4	-2	0	2	4
x	-10	-4	2	8	14
y	-5	-1	3	7	11

(b) $x = 3t + 2 \Rightarrow 3t = x - 2 \Rightarrow t = \frac{1}{3}x - \frac{21}{3}$, so

$y = 2t + 3 = 2\left(\frac{1}{3}x - \frac{2}{3}\right) + 3 = \frac{2}{3}x - + \frac{4}{3} + 3 \Rightarrow y = \frac{2}{3}x + \frac{5}{3}$

17. (a) $y = \sin t, \quad y = 1 - \cos t, \quad 0 \le t \le 2$



t	0	$\pi/2$	0	$3\pi/2$	2π
x	0	1	0	-1	0
y	0	1	2	1	0

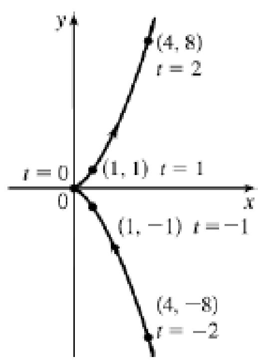
(b) $y = \sin t, y = 1 - \cos t$ [or $y - 1 = -\cos t$] $\Rightarrow$

$x^2 + (y-1)^2 = (\sin t)^2 + (-\cos t)^2 \Rightarrow$

$x^2 = (y-1)^2 = 1$. As t varies from 0 to 2π, the circle with center $(0,1)$ and radius 1 is traced out.

19. (a) $y = t^2, \quad y = t^3$

t	-2	-1	0	1	2
x	4	1	0	1	4
y	-8	-1	0	1	8



(b) $y = t^3 \Rightarrow t = \sqrt[3]{y} \Rightarrow x = t^2 = \left(\sqrt[3]{y}\right)^2 = y^{2/3}$

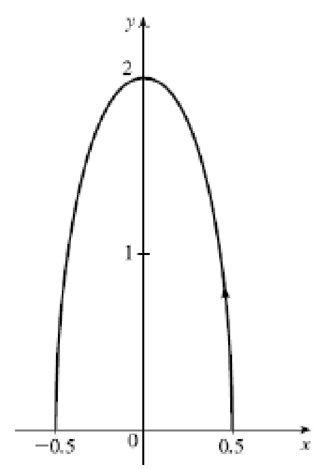
$t \in \mathbb{R}, y \in \mathbb{R}, x \ge 0.$

21. (a) $x = \frac{1}{2}\cos\theta, \ y = 2\sin\theta, \ 0 \le \theta \le \pi.$

$\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}y\right)^2 = \cos^2\theta + \sin^2\theta = 1 \Rightarrow 4x^2 + \frac{1}{4}y^2 = 1 \Rightarrow$

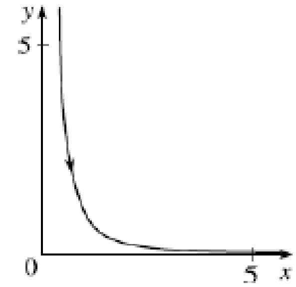
$\frac{x^2}{(1/2)^2} + \frac{y^2}{2^2} = 1$, which is an equation of an ellipse with x-intercepts $\pm\frac{1}{2}$ and y-intercepts ± 2. For $0 \le \theta \le \pi/2$, we have $\frac{1}{2} \ge x \ge 0$ and $0 \le y \le 2$. For $\pi/2 < \theta \le \pi$, we have $0 > x \ge -\frac{1}{2}$ and $2 > y \ge 0$. So the graph is the top half of an ellipse.

(b)



23. (a) $y = e^{-2t} = \left(e^{t}\right)^{-2} = x^{-2} = \dfrac{1}{x^2}$ for $x > 0$ since $x = e^{t}$.

(b)

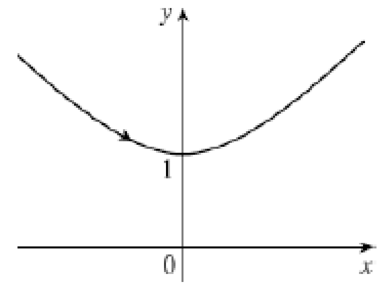


25. (a) $x = \sqrt{t+1} \Rightarrow x^2 = t+1 \Rightarrow t = x^2 - 1.$

$y = \sqrt{t-1} = \sqrt{(x^2-1)-1} = \sqrt{x^2-2}.$

The curve is the part of the hyperbola $x^2 - y^2 = 2,$ with $x \ge \sqrt{2}$ and $y \ge 0.$

(b)



27. $y = 2t+3 \Rightarrow \dfrac{y-3}{2} = t.$ $x = t^2+1 \Rightarrow x = \left(\dfrac{y-3}{2}\right)^2 + 1 = \dfrac{y^2-6y+9}{4} + 1 = \dfrac{y^2-6y+13}{4},$ option (**B**).

28. If $x = \ln t,$ then $e^x = t \Rightarrow y = 2t^2 = 2\left(e^x\right)^2 = 2e^{2x},$ choice (**C**).

29. Let $t = 1-x \Rightarrow 1 = t+x \Rightarrow x = 1-t.$ Then $y = \sqrt{\dfrac{x}{1-x}} = \dfrac{\sqrt{x}}{\sqrt{1-x}} = \dfrac{\sqrt{1-t}}{\sqrt{t}} = \sqrt{\dfrac{1-t}{t}},$ which is choice (**B**).

44. This curve is a quarter of a circle of radius 6, so the length of the path traveled by the particle is $\dfrac{1}{4}(2\pi \cdot 6) = \dfrac{12\pi}{4} = 3\pi.$

54. $x = t+1,\ y = 1-2t^2 \Rightarrow \dfrac{dx}{dt} = 1,\ \dfrac{dy}{dt} = -4t \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-4t}{1} = -4t.$ Critical points occur when

$\dfrac{dy}{dx} = 0 \Leftrightarrow -4t = 0 \Leftrightarrow t = 0.$ $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{\dfrac{dx}{dt}} = \dfrac{-4}{1} = -4 < 0.$ Because the curve is concave

down when $t = 0,$ the point when $t = 0$ is a maximum on the curve. At this point, $x = 0+1 = 1$ and $y = 1-2(0) = 1,$ so the x- and y-coordinates of this point are (**A**), $(1,1)$.

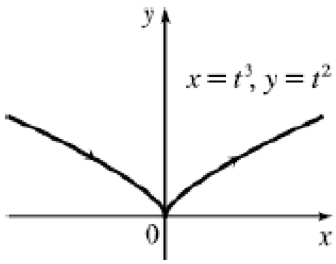
55. Looking ahead to the next section ,we know $L = \displaystyle\int_0^{\pi/3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$ In this case, we have

$x = \cos 2t \Rightarrow \dfrac{dx}{dt} = -2\sin 2t$ and $y = \sin 2t \Rightarrow \dfrac{dy}{dt} = 2\cos 2t.$

This means $\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 = (-2\sin 2t)^2 + (2\cos 2t)^2 = 4\left(\sin^2 2t + \cos^2 2t\right) = 4,$ so the length of the path is $L = \int_0^{\pi/3} \sqrt{4}\, dt = \int_0^{\pi/3} 2\, dt = 2t\Big|_0^{\pi/3} = \dfrac{2\pi}{3},$ option (**B**).

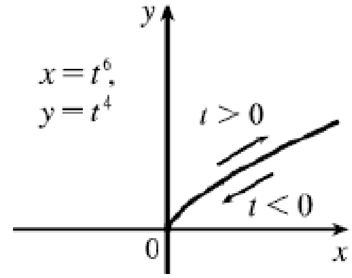
65. (a) $x = t^3 \Rightarrow t = x^{1/3}$, so $y = t^2 = x^{2/3}$

We get the entire curve $y = x^{2/3}$ traversed in a left to right direction.



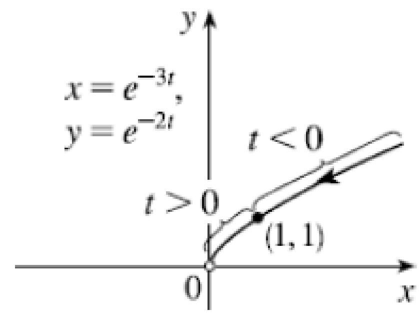
(b) $x = t^6 \Rightarrow t = x^{1/6}$, so $y = t^4 = x^{2/3}$

Since $x = t^6 \geq 0$, we only get the right half of the curve $y = x^{2/3}$.

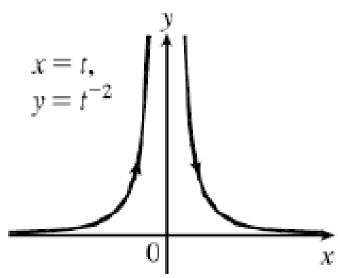


(c) $x = e^{-3t} = \left(e^{-t}\right)^3$, $\left[\text{so } e^{-t} = x^{1/3}\right]$,

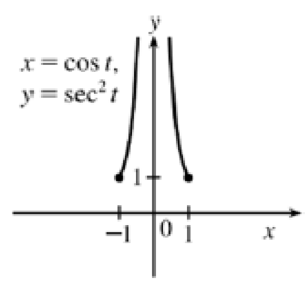
$y = e^{-2t} = \left(e^{-t}\right)^2 = (x^{1/3})^2 = x^{2/3}$. If $t < 0$, then x and y are both larger than 1. If $t > 0$, then x and y are between 0 and 1. Since $x > 0$ and $y > 0$, the curve never quite reaches the origin.



66. (a) $x = t$, so $y = t^{-2} = x^{-2}$. We get the entire curve $y = 1/x^2$ traversed in a left-to-right direction.



(b) $x = \cos t, y = \sec^2 t = \dfrac{1}{\cos^2 t} = \dfrac{1}{x^2}$. Because $\sec t \geq 1$, we only get the parts of the curve $y = 1/x^2$ with $y \geq 1$. We get the first quadrant portion of the curve when $x > 0$, that is, $\cos t > 0$, and we get the second quadrant portion of the curve when $x < 0$, that is, $\cos t < 0$.



(c) $x = e^t, y = e^{-2t} = \left(e^t\right)^{-2} = x^{-2}$. Since e^t and e^{-2t} are both positive, we only get the first quadrant portion of the curve $y = 1/x^2$.