p. 782: 7-11 odd, 17-25 odd, 35, 43-49 odd, 55, 60-62, 64-75

7. $x=\frac{t}{1+t},\ y=\sqrt{1+t} \Rightarrow \frac{dy}{dt}=\frac{1}{2}(1+t)^{-1/2}=\frac{1}{2\sqrt{1+t}},\ \frac{dx}{dt}=\frac{(1+t)(1)-t(1)}{(1+t)^2}=\frac{1}{(1+t)^2}$, and

$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{1/(2\sqrt{1+t})}{1/(1+t^2)}=\frac{(1+t)^2}{2\sqrt{1+t}}=\frac{1}{2}(1+t)^{3/2}$.

9. $x=t^3+1,\ y=t^4+t;\ t=-1.\ \frac{dy}{dt}=4t^3+1,\ \frac{dx}{dt}=3t^2$, and $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{4t^3+1}{3t^2}$. When $t=-1$, $(x,y)=(0,0)$ and $dy/dx=-3/3=-1$, so an equation of the tangent to the curve at the point corresponding to $t=-1$ is $y-0=-1(x-0)$, or $y=-x$.

11. $x=t\cos t,\ y=t\sin t;\ t=\pi.\ \frac{dy}{dt}=t\cos t+\sin t,\ \frac{dx}{dt}=t(-\sin t)+\cos t$, and

$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{t\cos t+\sin t}{-t\sin t+\cos t}$.

When $t=\pi$, $(x,y)=(-\pi,0)$ and $dy/dx=-\pi/(-1)=\pi$, so an equation of the tangent to the curve at the point corresponding to $t=\pi$ is $y-0=\pi(x-(-\pi))$, or $y=\pi x+\pi^2$.

17. $x=t^2+1,\ y=t^2+t \Rightarrow \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2t+1}{2t}=1+\frac{1}{2t} \Rightarrow \frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{dx/dt}=\frac{-1/(2t^2)}{2t}=-\frac{1}{4t^3}$. The curve is CU when $\frac{d^2y}{dt^2}>0$, that is, when $t<0$.

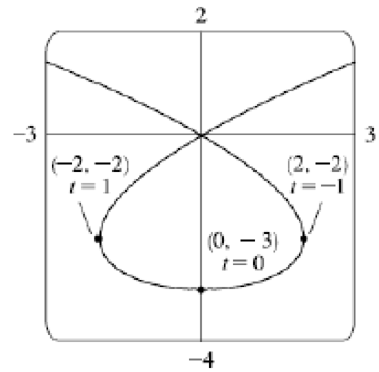
19. $x=e^t,\ y=te^{-t} \Rightarrow \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{-te^{-t}+e^{-t}}{e^t}=\frac{e^{-t}(1-t)}{e^t}=e^{-2t}(1-t) \Rightarrow$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{dx/dt}=\frac{e^{-2t}(-1)+(1-t)(-2e^{-2t})}{e^t}=\frac{e^{-2t}(-1-2+2t)}{e^t}=e^{-3t}(2t-3)$. The curve is CU when $\frac{d^2y}{dt^2}>0$, that is, when $t>\frac{3}{2}$.

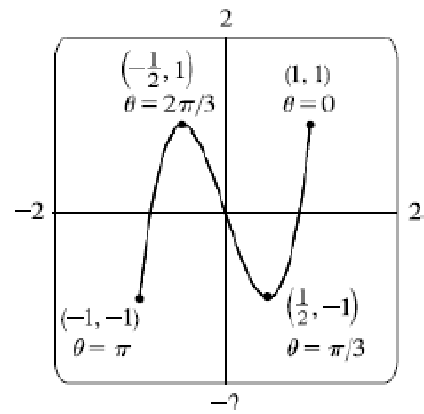
21. $x=t-\ln t,\ y=t+\ln t$ [note that $t>0$] $\Rightarrow \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{1+1/t}{1-1/t}=\frac{t+1}{t-1} \Rightarrow$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{dx/dt}=\frac{\frac{(t-1)(1)-(t+1)(1)}{(t-1)^2}}{(t-1)/t}=\frac{-2t}{(t-1)^3}$. The curve is CU when $\frac{d^2y}{dt^2}>0$, that is, when $0<t<1$.

23. $x = t^3 - 3t,\ y = t^3 - 3$. $\dfrac{dy}{dt} = 2t$, so $\dfrac{dy}{dt} = 0 \Leftrightarrow t = 0 \Leftrightarrow$ $(x, y) = (0, -3)$. $\dfrac{dx}{dt} = 3t^2 - 3 = 3(t+1)(t-1)$, so $\dfrac{dx}{dt} = 0 \Leftrightarrow$ $t = -1$ or $1 \Leftrightarrow (x, y) = (2, -2)$ or $(-2, -2)$. The curve has a horizontal tangent at $(0, -3)$ and vertical tangents at $(2, -2)$ and $(-2, -2)$.



25. $x = \cos\theta, y = \cos 3\theta$. The whole curve is traced out for $0 \le \theta \le \pi$. $\dfrac{dy}{d\theta} = -3\sin 3\theta$, so $\dfrac{dy}{d\theta} = 0 \Leftrightarrow \sin 3\theta = 0 \Leftrightarrow 3\theta = 0, \pi, 2\pi$, or $3\pi \Leftrightarrow$ $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$ or $\pi \Leftrightarrow (x, y) = (1,1), (\frac{1}{2}, -1), (-\frac{1}{2}, 1)$ or $(-1, -1)$. $\dfrac{dx}{d\theta} = -\sin\theta$, so $\dfrac{dx}{d\theta} = 0 \Leftrightarrow \sin\theta = 0 \Leftrightarrow \theta = 0$ or $\pi \Leftrightarrow (x, y) = (1,1)$ or $(-1, -1)$. Both $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$ equal 0 when $\theta = 0$, and π. To find the slope when $\theta = 0$, we find $\lim\limits_{\theta \to 0} \dfrac{dy}{dx} = \lim\limits_{\theta \to 0} \dfrac{-3\sin 3\theta}{-\sin\theta} \overset{H}{=} \lim\limits_{\theta \to 0} \dfrac{-9\cos 3\theta}{-\cos\theta} = 9$,



35. $x = 3t^2 + 1,\ y = t^3 - 1 \Rightarrow$ so $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2}{6t} = \dfrac{t}{2}$. The tangent has slope $\dfrac{1}{2}$ when $\dfrac{t}{2} = \dfrac{1}{2} \Leftrightarrow t = 1$, so the point is $(4, 0)$.

43. $x = t + e^{-t},\ y = t - e^{-t},\ 0 \le t \le 2$. $\dfrac{dx}{dt} = 1 - e^{-t}$ and $\dfrac{dy}{dt} = 1 + e^{-t}$, so

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(1 - e^{-t}\right)^2 + \left(1 + e^{-t}\right)^2 = 1 - 2e^{-t} + e^{-2t} + 1 + 2e^{-t} + e^{-2t} = 2 + 2e^{-2t}.$$ Thus,

$$L = \int_0^2 \sqrt{2 + 2e^{-2t}}\, dt \approx 3.142.$$

45. $x = t - 2\sin t,\ y = 1 - 2\cos t,\ 0 \le t \le 4\pi$. $\dfrac{dx}{dt} = 1 - 2\cos t$ and $\dfrac{dy}{dt} = 2\sin t$, so

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1 - 2\cos t)^2 + (2\sin t)^2 = 1 - 4\cos t + 4\cos^2 t + 4\sin^2 t = 5 - 4\cos t.$$ Thus,

$$L = \int_0^{4\pi} \sqrt{5 - 4\cos t}\, dt \approx 26.730.$$

47. $x = 1 + 3t^2,\ y = 4 + 2t^3,\ 0 \le t \le 1$. $dx/dt = 6t$ and $dy/dt = 6t^2$, so $(dx/dt)^2 + (dy/dt)^2 = 36t^2 + 36t^4$.

Thus, $L = \int_0^1 \sqrt{36t^2 + 36t^4}\, dt = \int_0^1 6t\sqrt{1 + t^2}\, dt = 6\int_1^2 \sqrt{u}\,(\frac{1}{2}\,du)$ $\quad [u = 1 + t^2,\ du = 2t\, dt]$

$$= 2\left[\tfrac{2}{3}u^{3/2}\right]_1^2 = 2\left(2^{3/2} - 1\right) = 2\left(2\sqrt{2} - 1\right)$$

49. $x = t\sin t,\ y = t\cos t,\ 0 \le t \le 1.\quad \dfrac{dx}{dt} = t\cos t + \sin t$ and $\dfrac{dy}{dt} = -t\sin t + \cos t$, so

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^2\cos^2 t + 2t\sin t\cos t + \sin^2 t + t^2\sin^2 t - 2t\sin t\cos t + \cos^2 t$$
$$= t^2(\cos^2 t + \sin^2 t) + \sin^2 t + \cos^2 t = t^2 + 1.$$

Thus, $L = \int_0^1 \sqrt{t^2+1}\,dt = \left[\tfrac{1}{2}t\sqrt{t^2+1} + \tfrac{1}{2}\ln\left(t + \sqrt{t^2+1}\right)\right]_0^1 = \tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\ln\left(1+\sqrt{2}\right).$

55. $x(t) = t^2 \Rightarrow x'(t) = 2t,\ y(t) = 2e^{2t} \Rightarrow y'(t) = 4e^{2t} \Rightarrow$

$L = \int_a^b \sqrt{x'(t) + y'(t)}dt = \int_0^{\ln 2} \sqrt{(2t)^2 + (4e^{2t})^2}\,dt = \int_0^{\ln 2} \sqrt{4t^2 + 16e^{4t}}\,dt$, choice (**D**).

60. $\mathbf{r}(t) = \left\langle t^3, \sin\left(\frac{\pi}{t}\right)\right\rangle \Rightarrow \mathbf{v}(t) = \left\langle 3t^2, -\frac{\pi}{t^2}\cos\left(\frac{\pi}{t}\right)\right\rangle \Rightarrow \mathbf{v}(2) = \left\langle 3(2)^2, -\frac{\pi}{2^2}\cos\left(\frac{\pi}{2}\right)\right\rangle = \langle 12, 0\rangle$, choice (**A**).

61. $\dfrac{dy}{dt} = \dfrac{t^2+1}{t-1}\cdot\dfrac{dx}{dt} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{t^2+1}{t-1} \Rightarrow \left.\dfrac{dy}{dt}\right|_{t=3} = \dfrac{3^2+1}{3-1} = \dfrac{10}{2} = 5$, (**A**).

62. speed $= |\mathbf{v}(t)| = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(6t)^2 + 2^2} = \sqrt{36t^2+4}.$

$24 = \sqrt{36t^2+4} \Leftrightarrow 24^2 = 36t^2 + 4 \Leftrightarrow 576 - 4 = 36t^2 \Leftrightarrow \frac{572}{36} = \frac{143}{9} = t^2 \Leftrightarrow t = \frac{\sqrt{143}}{3}$, choice (**C**).

64. $x = \cos t \Rightarrow \dfrac{dx}{dt} = -\sin t,\ y = \cos 2t \Rightarrow \dfrac{dy}{dt} = -2\sin 2t.$

$L = \int_0^{\pi} \sqrt{(-\sin t)^2 + (-2\sin 2t)^2}\,dt = \int_0^{\pi} \sqrt{\sin^2 t + 4\sin^2(2t)}\,dt \overset{\text{CAS}}{=} 4.647$, choice (**B**).

65. Speed $= \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2} = \sqrt{\left(4 + \cos t^2\right)^2 + \left(\dfrac{dy}{dt}\right)^2}$. When $t = 1$,

speed $= \sqrt{(4+\cos 1)^2 + 16} = 6.051$, which is option (**C**).

66. The curve has a horizontal tangent when $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = 0 \Leftrightarrow \dfrac{dy}{dt} = 0$ and $\dfrac{dx}{dt} \ne 0.$

$y = \dfrac{2t^2-8}{t^3-1} \Rightarrow \dfrac{dy}{dt} = \dfrac{(t^3-1)(4t) - (2t^2-8)(3t^2)}{(t^3-1)^2} = \dfrac{-2t(t^3-12t+2)}{(t^3-1)^2}.\ \dfrac{dy}{dt} = 0 \Leftrightarrow t = 0$, and when $t = 0$,

$\dfrac{dx}{dt} = \dfrac{(t+2)(4) - (4t)(1)}{(t+2)^2} = \dfrac{8}{(t+2)^2} \ne 0.$

Therefore, the only horizontal tangent line occurs when $t = 0 \Rightarrow y = \dfrac{2\cdot 0^2 - 8}{0^3 - 1} = \dfrac{-8}{-1} = 8$, (**D**).

67. The region R is traced for $0 \le t \le \frac{\pi}{2}$.

$x = \cos^3 t \Rightarrow x' = 3\cos^2 t\cdot(-\sin t),\ y = \sin^3 t \Rightarrow y' = 3\sin^2 t\cdot(\cos t).$

$$P=\int_a^b \sqrt{(x')^2+(y')^2}\,dt=\int_0^{\pi/2}\sqrt{(3\cos^2 t\cdot(-\sin t))^2+(3\sin^2 t\cdot(\cos t))^2}\,dt$$

$$=3\int_0^{\pi/2}\sqrt{\left[\cos^4 t\sin^2 t+\sin^4 t\cos^2 t\right]}\,dt=3\int_0^{\pi/2}\sqrt{\left[\cos^2 t\sin^2 t\left(\cos^2 t+\sin^2 t\right)\right]}\,dt$$

$$=3\int_0^{\pi/2}\cos t\sin t\,dt=\tfrac{3}{2}\sin^2 t\Big|_0^{\pi/2}=1.5,\text{ option } (\mathbf{B}).$$

68. $x=t^2+1\Rightarrow \dfrac{dx}{dt}=2t,\ y=t^3\Rightarrow \dfrac{dy}{dt}=3t^2.$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3t^2}{2t}=\tfrac{3}{2}t.\quad \frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{dx/dt}=\frac{\frac{3}{2}}{2t}=\frac{3}{4t},\text{ choice } (\mathbf{B}).$$

69. (a) $\mathbf{a}(t)=\mathbf{v}'(t)=\langle x''(t),y''(t)\rangle=\left\langle \dfrac{(1+t^2)(1)-t(2t)}{(1+t^2)^2},-2t^{-2}\right\rangle=\left\langle \dfrac{1-t^2}{(1+t^2)^2},-2t^{-2}\right\rangle.$

$$\mathbf{a}(1)=\left\langle \frac{1-(1)^2}{(1+1^2)^2},\frac{-2}{(1)^2}\right\rangle=\langle 0,-2\rangle.$$

(b) $x(t)=\dfrac{\ln 2}{2}+\displaystyle\int_1^t \frac{s}{1+s^2}ds=\frac{\ln 2}{2}+\frac{1}{2}\left[\ln(1+s^2)\right]_1^t=\frac{\ln 2}{2}+\frac{\ln(1+t^2)-\ln 2}{2}=\frac{\ln(1+t^2)}{2}\Rightarrow x(2)=\frac{\ln 5}{2}$

$$y(t)=2+\int_1^t \frac{2}{s}ds=2+2\left[\ln s\right]_1^t=2+2(\ln t-0)=2+2\ln t\Rightarrow y(2)=2+2\ln 2$$

The particle's position at time $t=2$ is $\langle \tfrac{1}{2}\ln 5, 2(1+\ln 2)\rangle$.

(c) $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{2/t}{t/(1+t^2)}=\dfrac{2}{t}\cdot\dfrac{1+t^2}{t}=2\left(\dfrac{1+t^2}{t^2}\right)=2\left(1/t^2+1\right).$

Therefore, $\displaystyle\lim_{t\to\infty}\frac{dy}{dx}=2\cdot\lim_{t\to\infty}\left(1/t^2+1\right)=2.$

70. (a) $\mathbf{a}(t)=\mathbf{v}'(t)=\left\langle \dfrac{3(2+t)^2}{6+(2+t)^3},6-4t\right\rangle\Rightarrow \mathbf{a}(1)=\left\langle \dfrac{3(2+1)^2}{6+(2+1)^3},6-4(1)\right\rangle=\langle \tfrac{27}{33},2\rangle$

Speed $=|\mathbf{v}(t)|=\sqrt{\left[\ln(6+(2+t)^3\right]^2+(6t-2t^2)^2}\Rightarrow$

$$|\mathbf{v}(1)|=\sqrt{\left[\ln(6+(3)^3)\right]^2+(6-2)^2}=\sqrt{\left[\ln(33)\right]^2+16}\approx 5.313.$$

(b) $x(1)=5+\int_0^2 \ln\left[6+(2+t)^3\right]dt\approx 5+6.955=11.955.$

$y(2)=9+\int_1^2(6t-2t^2)\,dt=9+\left[3t^2-\tfrac{2}{3}t^3\right]_1^2=9+\left(12-\tfrac{16}{3}\right)-(0-0)=\tfrac{47}{3}.$ Thus, $P\approx\langle 1.955,\tfrac{47}{3}\rangle.$

(c) The particle is at rest when $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=0\Rightarrow\dfrac{dy}{dt}=0.$ (Note that $dx/dt\neq 0$ for any $t\geq 0$.)

$$\frac{dy}{dt}=0\Leftrightarrow 6t-2t^2=2t(3-t)=0\Leftrightarrow t=0\text{ or } t=3.$$

(d) The slope of the tangent line is $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{6t-2t^2}{\ln[6+(2+t)^2]}$. At P, this slope is

$\dfrac{6\cdot 2-2\cdot 2^2}{\ln[6+3^3]}=\dfrac{4}{\ln 33}$, so the equation of the line tangent to the curve at P is $y-\frac{47}{3}=\frac{4}{\ln 33}(x-11.955)$.

71. (a) $\mathbf{v}(t)=\langle x'(t),y'(t)\rangle \Rightarrow \mathbf{v}(1)=\langle e^{\cos(1)},2\cos(1)\rangle \approx \langle 1.717,1.081\rangle$.

$\mathbf{a}(t)=\mathbf{v}'(t)=\langle -2\sin t\cdot e^{\cos t},-2\sin t\rangle \Rightarrow \mathbf{a}(1)=\langle -2\sin(1)\cdot e^{\cos 1},-2\sin(1)\rangle \approx \langle -2.889,-1.683\rangle$.

(b) speed = $\sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}=\sqrt{\left(e^{\cos t}\right)^2+(2\cos t)^2}$.

$1.5^2=2.25=\left(e^{\cos t}\right)^2+4\cos^2 t \Leftrightarrow t\approx 1.254, 2.358$. The first time t for which the speed of the particle is 1.5 is $t\approx 1.254$.

(c) At the highest point, the y-coordinate of the particle is at its maximum value, so

$\dfrac{dy}{dt}=0 \Leftrightarrow 2\cos t=0 \Leftrightarrow \cos t=0 \Leftrightarrow t=\frac{\pi}{2}$. $y(0)=y(\pi)=0$, $y\left(\frac{\pi}{2}\right)=2\cdot 1=2$, so this point is a maximum. At this point, $x\left(\frac{\pi}{2}\right)=1+\int_0^{\pi/2} e^{\cos t}\,dt \overset{\text{CAS}}{=} 1+3.104=4.104$. Thus, the coordinates of the particle at its highest point are roughly $\langle 4.104,2\rangle$.

(d) Because the particle is at its highest point when $t=\frac{\pi}{2}$, we have already determined that the particle is at position $\langle 4.104,2\rangle$ when $t=\frac{\pi}{2}$.

(e) distance traveled = $\displaystyle\int_0^{\pi}\sqrt{\left(e^{\cos t}\right)^2+(2\sin t)^2}\,dt\approx 6.035$.

(f) $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{-2\cos t}{e^{\cos t}}=\dfrac{1}{2} \Leftrightarrow 4\cos t=e^{\cos t} \Leftrightarrow t=a\approx 1.20531$.

$y(a)\approx 2\sin(a)\approx 1.868$. $x(a)=1+\int_0^a e^{\cos t}\,dt \overset{\text{CAS}}{\approx} 1+2.664=3.664$.

At this point, the coordinates of the particle are $(3.664,1.868)$.

(g) $\dfrac{d^2y}{dx^2}=\dfrac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{\frac{dx}{dt}}=\dfrac{-2\sin t}{e^{\cos t}}$. When $t=1$, $\dfrac{d^2y}{dx^2}=\dfrac{-2\sin 1}{e^{\cos 1}}\approx -0.980<0$, so the curve is concave down at this point.

72. (a) $\mathbf{a}(t)=\mathbf{v}(t)=\dfrac{3}{2\cdot\sqrt{t+4}} \Rightarrow \mathbf{a}(3)=\dfrac{3}{2\cdot\sqrt{3+4}}=\dfrac{3}{2\sqrt{7}}=\dfrac{3\sqrt{7}}{14}\approx 0.567$

(b) $x(5)=12+\int_0^5 3\sqrt{t+4}\,dt=12+\left[2(t+4)^{3/2}\right]_0^5=12+2\left(9^{3/2}-4^{3/2}\right)=12+2(27-8)=12+38=50$

(c) $\mathbf{v}_y(t)=0+\int_5^t 10\,dy=10t-50=10(t-5)$

(d) speed $=|\mathbf{v}(t)|=\sqrt{\left(3\sqrt{t+4}\right)^2+(10t-50)^2}=\sqrt{100t^2-991t+2536}$ $\quad |\mathbf{v}(7)|=\sqrt{49}\approx 22.338$

73. (a) Speed at time $t=2$ is $\sqrt{\left(e^{2^2}\right)^2+\left(\sin\left(2^2\right)\right)^2}\approx 54.603$

$\mathbf{a}(t)=\mathbf{v}'(t)=\langle 2te^{t^2},2t\cos(t^2)\rangle \Rightarrow \mathbf{a}(2)=\langle 4e^4,4\cos(4)\rangle\approx\langle 218.393,-2.615\rangle$

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{dy/dt}{dx/dt}\right|_{t=2} = \frac{\sin(4)}{e^4} \approx -0.91386$

$x(2) = 0 + \int_0^2 e^{t^2}\,dt \approx 16.453,\ \ y(2) = -4 + \int_0^2 \sin(t^2)\,dt \approx -3.195.$

Thus, the equation of the line tangent to the curve when $t = 2$ is $y = \frac{\sin 4}{e^4}(x - 16.453) - 3.195.$

(c) distance traveled $= \int_0^2 \sqrt{(e^{t^2})^2 + \left(\sin(t^2)\right)^2}\,dt \approx 16.545$

74. (a) $y(4) = 0.01(4^3 - 22(4^2) + 120(4)) = 1.92$

$x(4) = 1 + \int_2^4 \sin\left(\frac{\pi(t-2)^2}{36}\right)dt \approx 1 + 0.231 = 1.231.$ Thus at time $t = 4,$ the particle's coordinates are approximately $(1.231, 1.92).$

(b) The speed is equal to 1 when $1 = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \Rightarrow t = a \approx 0.51460$ (and a graph confirms this is the first time). At this time, $\mathbf{a}(b) = \mathbf{v}'(b) = \langle -0.254, 0.982 \rangle.$

(c) The tangent line is vertical when

$\frac{dx}{dt} = 0 \Leftrightarrow \sin\left(\frac{\pi(t-2)^2}{36}\right) = 0 \Leftrightarrow \frac{\pi(t-2)^2}{36} = 0, \pi, 2\pi,$ or $3\pi, \Leftrightarrow t = 2$ or $8.$

(d) The maximum height of the particle occurs when the tangent line is horizontal, that is, when

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0 \Leftrightarrow \frac{dy}{dt} = 0 \Leftrightarrow 0.01\left(3t^2 - 44t + 120\right) = 0 \Leftrightarrow 3t^2 - 44t + 120 = 0 \Leftrightarrow t \approx 3.621,$ or $11.045.$

$y(3.621) \approx 1.935 < 2,$ and $y(11.045) \approx -0.110 < 2,$ so the particle never exceeds a height of 2.

(e) $P = \int_0^{10} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 6.874$

(f) Average speed $= \frac{1}{10-0}\int_0^{10} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \approx \frac{1}{10}(6.87387) = 0.687$

75. (a) $|\mathbf{v}'(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}.\ |\mathbf{v}'(1)| = \sqrt{217} \approx 14.731.$

(b) $L = \int_0^5 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \overset{\text{CAS}}{\approx} 59.725$

(c) $x(t) = 4 + \int_0^2 (-2 + e^{-t^2+1})\,dt \approx 2.40,\ \ y(t) = 5 + \int_0^2 3\sqrt{25 - t^2}\,dt \approx 34.180 \Rightarrow$ At time $t = 2,$ the particle's position is $\langle 2.40, 34.180 \rangle,$

(d) $\frac{dx}{dt} = 0 \Leftrightarrow -2 + e^{-t^2+1} = 0 \Leftrightarrow 2 = e^{-t^2+1} \Leftrightarrow \ln 2 = -t^2 + 1 \Leftrightarrow t^2 = 1 - \ln 2 \Rightarrow t = b = \sqrt{1 - \ln 2}.$

At time $t = b,\ \mathbf{a}(b) = \mathbf{v}'(b) = \langle -2.216, -0.334 \rangle.$