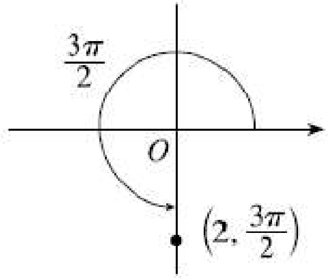
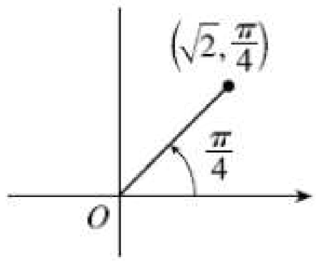
p. 800: 18-22, 23-27 odd, 31-42, 45-55 odd, 70, 71-81 odd, 90, 99-100, 102-103

18. (a)



$x = 2\cos\frac{3\pi}{2} = 2(0) = 0$ and $y = 2\sin\frac{3\pi}{2} = 2(-1) = -2$ give us the Cartesian coordinates $(0,-2)$.

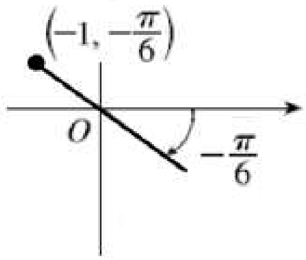
(b)



$x = \sqrt{2}\cos\frac{\pi}{4} = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 1$ and

$y = \sqrt{2}\sin\frac{\pi}{4} = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 1$

(c)

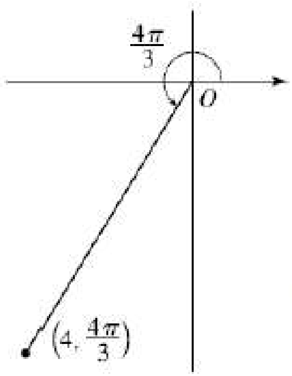


$x = -1\cdot\cos\left(-\frac{\pi}{6}\right) = -1\cdot\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$ and

$y = -1\cdot\sin\left(-\frac{\pi}{6}\right) = -1\cdot\left(-\frac{1}{2}\right) = \frac{1}{2}$

give us the Cartesian coordinates $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

19. (a)

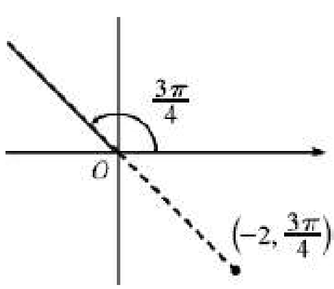


$x = 4\cos\frac{4\pi}{3} = 4\left(-\frac{1}{2}\right) = -2$ and

$y = 4\sin\frac{4\pi}{3} = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

give us the Cartesian coordinates $\left(-2,-2\sqrt{3}\right)$.

(b)

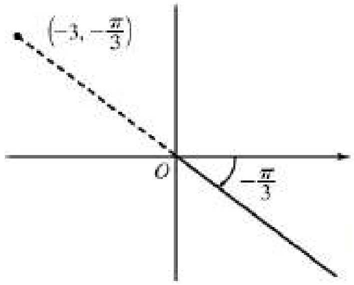


$x = -2\cos\frac{3\pi}{4} = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$ and

$y = -2\sin\frac{3\pi}{4} = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

give us the Cartesian coordinates $\left(\sqrt{2},-\sqrt{2}\right)$.

(c)



$x=-3\cos\left(-\frac{\pi}{3}\right)=-3\left(\frac{1}{2}\right)=-\frac{3}{2}$ and

$y=-3\sin\left(-\frac{\pi}{3}\right)=-3\left(-\frac{\sqrt{3}}{2}\right)=\frac{3\sqrt{3}}{2}$

give us the Cartesian coordinates $\left(-\frac{\sqrt{3}}{2},\frac{3\sqrt{3}}{2}\right)$.

20. (a) $x=-4$ and $y=4 \Rightarrow r=\sqrt{(-4)^2+4^2}=4\sqrt{2}$ and $\tan\theta=\frac{4}{-4}=-1$ $\left[\theta=-\frac{\pi}{4}+n\pi\right]$. Since $(-4,4)$ is in the second quadrant, the polar coordinates are (i) $\left(4\sqrt{2},\frac{3\pi}{4}\right)$ and (ii) $\left(-4\sqrt{2},\frac{7\pi}{4}\right)$.

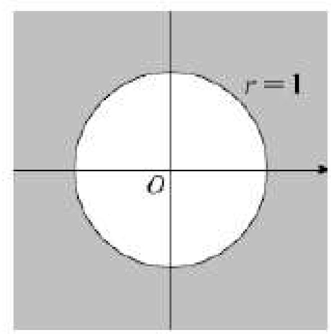
(b) $x=3$ and $y=3\sqrt{3} \Rightarrow r=\sqrt{3^2+(3\sqrt{3})^2}=\sqrt{9+27}=6$ and $\tan\theta=\frac{3\sqrt{3}}{3}=\sqrt{3}$ $\left[\theta=\frac{\pi}{3}+n\pi\right]$. Since $\left(3,3\sqrt{3}\right)$ is in the first quadrant, the polar coordinates are $\left(6,\frac{\pi}{3}\right)$ and (ii) $\left(-6,\frac{4\pi}{3}\right)$.

21. (a) $x=\sqrt{3}$ and $y=-1 \Rightarrow r=\sqrt{(\sqrt{3})^2+(-1)^2}=2$ and $\tan\theta=\frac{-1}{\sqrt{3}}$ $\left[\theta=-\frac{\pi}{6}+n\pi\right]$. Since $\left(\sqrt{3},-1\right)$ is in the fourth quadrant, the polar coordinates are (i) $\left(2,\frac{11\pi}{6}\right)$ and (ii) $\left(-2,\frac{5\pi}{6}\right)$.

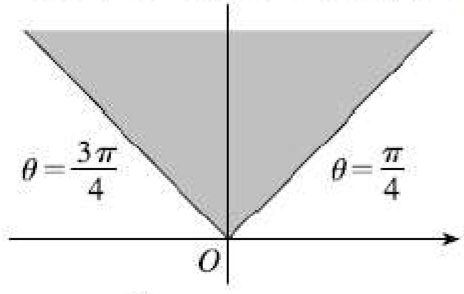
(b) $x=-6$ and $y=0 \Rightarrow r=\sqrt{(-6)^2+0^2}=6$ and $\tan\theta=\frac{0}{-6}=0$ $[\theta=n\pi]$. Since $(-6,0)$ is on the negative x-axis, the polar coordinates are $(6,\pi)$ and (ii) $(-6,0)$.

22. The point $\left(1,-\frac{2\pi}{3}\right)$, (C), is not the same as $\left(1,\frac{2\pi}{3}\right)$.

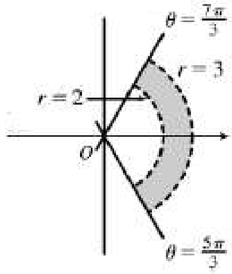
23. $r\geq 1$. The curve represents a circle with center O and radius 1. So $r \geq 1$ represents the region on or outside the circle. Note that θ can take on any value.



25. $r\geq 0,\ \pi/4\leq\theta\leq 3\pi/4$ $\theta=k$ represents a line through O.



27. $2<r<3,\ \frac{5\pi}{3}<\theta<\frac{7\pi}{3}$



31. $r^2 = 5 \Leftrightarrow x^2 + y^2 = 5$ is a circle of radius $\sqrt{5}$ centered at the origin.

32. $r = \sec\theta \Leftrightarrow \dfrac{r}{\sec\theta} = 4 \Leftrightarrow r\cos\theta = 4 \Leftrightarrow x = 4,$ a vertical line.

33. $r = 5\cos\theta \Rightarrow r^2 = 5r\cos\theta \Leftrightarrow x^2 + y^2 = 5x \Leftrightarrow x^2 - 5x + \frac{25}{4} + y^2 = \frac{25}{4} \Leftrightarrow \left(x - \frac{5}{2}\right)^2 + y^2 = \frac{25}{4},$
$\left(x - \frac{5}{2}\right)^2 + y^2 = \frac{25}{4}$, a circle of radius $\frac{5}{2}$ centered at $\left(\frac{5}{2}, 0\right)$. The first two equations are actually equivalent since $r^2 = 5r\cos\theta \Rightarrow r(r - 5\cos\theta) = 0 \Rightarrow r = 0$ or $r = 5\cos\theta$. But $r = 5\cos\theta$ gives the point $r = 0$ (the pole) when $\theta = 0$. Thus, the equation $r = 5\cos\theta$ is equivalent to the compound condition $(r = 0 \text{ or } r = 5\cos\theta)$.

34. $\theta = \dfrac{\pi}{3} \Rightarrow \tan\theta = \tan\dfrac{\pi}{3} \Rightarrow \dfrac{y}{x} = \sqrt{3} \Leftrightarrow y = \sqrt{3}x,$ a line through the origin.

35. $r^2\cos 2\theta = 1 \Leftrightarrow r^2(\cos^2\theta - \sin^2\theta) = 1 \Leftrightarrow (r\cos\theta)^2 - (r\sin\theta)^2 = 1 \Leftrightarrow x^2 - y^2 = 1,$ a hyperbola centered at the origin with foci on the x-axis.

36. $r^2\sin 2\theta = 1 \Leftrightarrow r^2(2\sin\theta\cos\theta) = 1 \Leftrightarrow 2(r\cos\theta)(r\sin\theta) = 1 \Leftrightarrow 2xy = 1 \Leftrightarrow xy = \frac{1}{2},$ a hyperbola centered at the origin with foci on the line $y = x$.

37. $y = 2 \Leftrightarrow r\sin\theta = 2 \Leftrightarrow r = \dfrac{2}{\sin\theta} \Leftrightarrow r = 2\csc\theta$

38. $y = x \Rightarrow \dfrac{y}{x} = 1\ [x \neq 0] \Rightarrow \tan\theta = 1 \Rightarrow \theta = \tan^{-1}1 \Rightarrow \theta = \dfrac{\pi}{4}$ or $\theta = \dfrac{5\pi}{4}$ [either includes the pole]

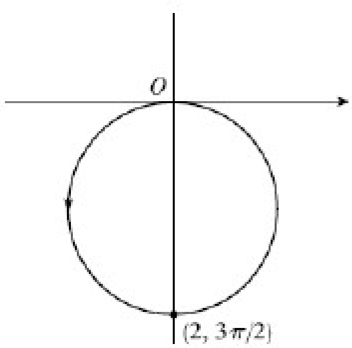
39. $y = 1 + 3x \Leftrightarrow r\sin\theta = 1 + 3r\cos\theta \Leftrightarrow r\sin\theta - 3r\cos\theta = 1 \Leftrightarrow r(\sin\theta - 3\cos\theta) = 1 \Leftrightarrow$
$r = \dfrac{1}{2\sin\theta - 3\cos\theta}$

40. $4y^2 = x \Leftrightarrow 4(r\sin\theta)^2 = r\cos\theta \Leftrightarrow 4r^2\sin^2\theta - r\cos\theta = 0 \Leftrightarrow r(4r\sin^2\theta - \cos\theta) = 0 \Leftrightarrow r = 0$ or
$r = \dfrac{\cos\theta}{4\sin^2\theta} \Leftrightarrow r = 0$ or $r = \frac{1}{4}\cot\theta\csc\theta$. $r = 0$ is included in $r = \frac{1}{4}\cot\theta\csc\theta$ when $\theta = \frac{\pi}{2}$, so the curve is represented by the single equation $r = \frac{1}{4}\cot\theta\csc\theta$.

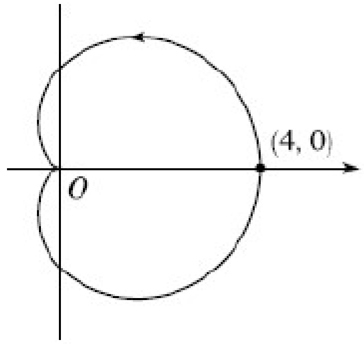
41. $x^2 + y^2 = 2cx \Leftrightarrow r^2 = 2cr\cos\theta \Leftrightarrow r^2 - 2cr\cos\theta = 0 \Leftrightarrow r(r - 2c\cos\theta) = 0 \Leftrightarrow r = 0$ or $r = 2c\cos\theta$.
$r = 0$ is included in $r = 2c\cos\theta$ when $\theta = \frac{\pi}{2} + n\pi$, so the curve is represented by the single equation $r = 2c\cos\theta$.

42. $x^2 - y^2 = 4 \Leftrightarrow (r\cos\theta)^2 - (r\sin\theta)^2 = 4 \Leftrightarrow r^2\cos^2\theta - r^2\sin^2\theta = 4 \Leftrightarrow r^2(\cos^2\theta - \sin^2\theta) = 4 \Leftrightarrow$
$r^2\cos 2\theta = 4$

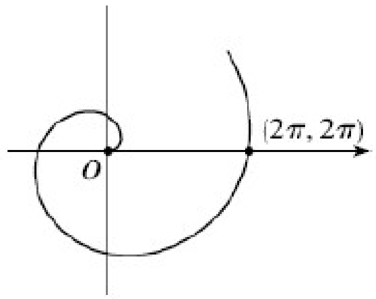
45. $r = -2\sin\theta$



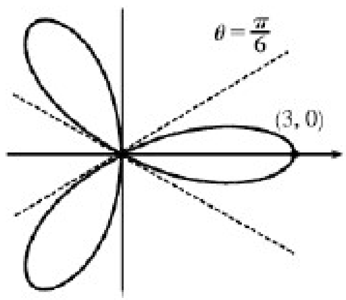
47. $r = 2(1+\cos\theta)$



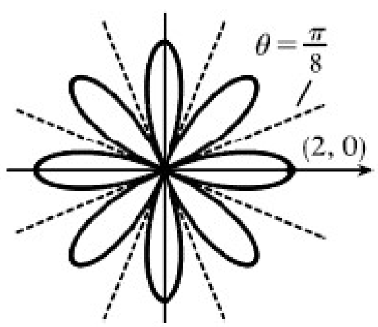
49. $r = \theta, \quad \theta \geq 0$



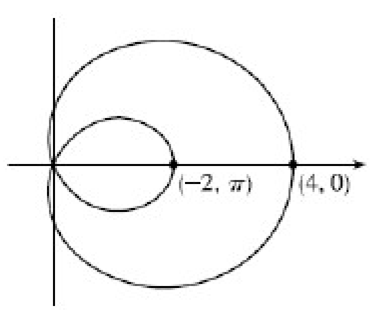
51. $r = 3\cos 3\theta$



53. $r = 2\cos 4\theta$



55. $r = 1+3\cos\theta$



70. The slope of the tangent line is $\frac{dy}{dx} = \frac{(f(\theta)\cdot\sin\theta)'}{(f(\theta)\cdot\cos\theta)'} = \frac{f(\theta)\cdot\cos\theta + \sin\theta\cdot f'(\theta)}{f(\theta)\cdot(-\sin\theta) + \cos\theta\cdot f'(\theta)} \Rightarrow$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{f\left(\frac{\pi}{2}\right)\cdot\cos\theta + \sin\left(\frac{\pi}{2}\right)\cdot f'\left(\frac{\pi}{2}\right)}{f\left(\frac{\pi}{2}\right)\cdot\left(-\sin\left(\frac{\pi}{2}\right)\right) + \cos\left(\frac{\pi}{2}\right)\cdot f'\left(\frac{\pi}{2}\right)} = \frac{10\cdot 0 + 1\cdot 4}{10(-1) + 0\cdot 4} = \frac{4}{-10} = -\frac{2}{5},$$ choice (**B**).

71. $r = 2\cos\theta \Rightarrow x = r\cos\theta = 2\cos^2\theta,\ y = r\sin\theta = 2\sin\theta\cos\theta = \sin 2\theta \Rightarrow$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos 2\theta}{2\cdot 2\cos\theta(-\sin\theta)} = \frac{\cos 2\theta}{-\sin 2\theta} = -\cot 2\theta$$

When $\theta = \frac{\pi}{3}$, $\frac{dy}{dx} = -\cot\left(2\cdot\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{\sqrt{3}}$.

73. $r = 1/\theta \Rightarrow x = r\cos\theta = (\cos\theta)/\theta,\ y = r\sin\theta = (\sin\theta)/\theta \Rightarrow$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sin\theta(-1/\theta^2) + (1/\theta)\cos\theta}{\cos\theta(-1/\theta^2) - (1/\theta)\sin\theta}\cdot\frac{\theta^2}{\theta^2} = \frac{-\sin\theta + \theta\cos\theta}{-\cos\theta - \theta\sin\theta}$$

When $\theta = \pi$, $\frac{dy}{dx} = \frac{-0 + \pi(-1)}{-(-1) - \pi(0)} = \frac{-\pi}{1} = -\pi$.

75. $r = \cos 3\theta \rightarrow x = r\cos\theta = \cos 3\theta\cos\theta,\ y = r\sin\theta = \cos 3\theta\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos 3\theta\cos\theta + \sin\theta(-3\sin 3\theta)}{\cos 3\theta\,(-\sin\theta) + \cos\theta(-3\sin 3\theta)}$$

When $\theta = \frac{\pi}{4}$:

$$\frac{dy}{dx} = \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)(-3)\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)(-3)\left(\frac{\sqrt{2}}{2}\right)} = \frac{-\frac{1}{2} - \frac{3}{2}}{\frac{1}{2} - \frac{3}{2}} = \frac{-2}{-1} = 2$$

77. $\frac{dy}{dx} = \frac{\cos 2\theta(\cos\theta) + \sin\theta(-2\sin 2\theta)}{\cos 2\theta(-\sin\theta) + \cos\theta(-2\sin 2\theta)} \Rightarrow$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{\cos\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{4}\right) - 2\sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{2}\right)\left(-\sin\left(\frac{\pi}{4}\right)\right) - 2\sin\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{4}\right)} = \frac{0\cdot\frac{\sqrt{2}}{2} - 2\cdot 1\cdot\frac{\sqrt{2}}{2}}{0\cdot\left(-\frac{\sqrt{2}}{2}\right) - 2\cdot 1\cdot\frac{\sqrt{2}}{2}} = 1,$$ choice (**D**).

79. $r = 1 - \sin\theta \Rightarrow x = r\cos\theta = \cos\theta(1-\sin\theta),\ y = r\sin\theta = \sin\theta(1-\sin\theta) \Rightarrow$

$\frac{dy}{d\theta} = \sin\theta(-\cos\theta) + (1-\sin\theta)\cos\theta = \cos\theta(1-2\sin\theta) = 0 \Rightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$,
or $\frac{3\pi}{2} \Rightarrow$ horizontal tangent at $\left(\frac{1}{2}, \frac{\pi}{6}\right), \left(\frac{1}{2}, \frac{5\pi}{6}\right)$ and $\left(2, \frac{3\pi}{2}\right)$.

$\frac{dx}{d\theta} = \cos\theta(-\cos\theta) + (1-\sin\theta)(-\sin\theta) = -\cos^2\theta - \sin\theta + \sin^2\theta = 2\sin^2\theta - \sin\theta - 1$
$= (2\sin\theta + 1)(\sin\theta - 1) = 0 \Rightarrow \sin\theta = -\frac{1}{2}$ or $1 \Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$, or $\frac{\pi}{2} \Rightarrow$ vertical tangents at $\left(\frac{3}{2}, \frac{7\pi}{6}\right), \left(\frac{3}{2}, \frac{11\pi}{6}\right)$ and $\left(0, \frac{\pi}{2}\right)$. Note that the tangent is vertical, not horizontal when $\theta = \frac{\pi}{2}$, since

$$\lim_{\theta\to(\pi/2)^-} \frac{dy/d\theta}{dx/d\theta} = \lim_{\theta\to(\pi/2)^-} \frac{\cos\theta(1-2\sin\theta)}{(2\sin\theta+1)(\sin\theta-1)} = \infty \text{ and } \lim_{\theta\to(\pi/2)^+} \frac{dy/d\theta}{dx/d\theta} = -\infty.$$

81. $r=e^{\theta} \Rightarrow x=r\cos\theta=e^{\theta}\cos\theta,\ y=r\sin\theta=e^{\theta}\sin\theta \Rightarrow$

$\frac{dy}{d\theta}=e^{\theta}\sin\theta+e^{\theta}\cos\theta=e^{\theta}(\sin\theta+\cos\theta)=0 \Rightarrow \sin\theta=-\cos\theta \Rightarrow \tan\theta=-1 \Rightarrow$

$\theta=-\frac{1}{4}\pi+n\pi$ [n any integer] $\Rightarrow$ horizontal tangents at $\left(e^{\pi(n-1)/4},\pi(n-\frac{1}{4})\right)$.

$\frac{dx}{d\theta}=e^{\theta}\cos\theta-e^{\theta}\sin\theta=e^{\theta}(\cos\theta-\sin\theta)=0 \Rightarrow \sin\theta=\cos\theta \Rightarrow \tan\theta=1 \Rightarrow$

$\theta=\frac{1}{4}\pi+n\pi$ [n any integer] $\Rightarrow$ vertical tangents at $\left(e^{\pi(n+1)/4},\pi(n+\frac{1}{4})\right)$.

90. Because $\sin\theta=\cos\left(\theta-\frac{\pi}{2}\right)$, the equation $r=\cos\left(\theta-\frac{\pi}{2}\right)+1=1+\cos\left(\theta-\frac{\pi}{2}\right)$ is equivalent to $r=1+\sin\theta$. Therefore, choice (**A**) is correct.

99. $x=\dfrac{5\cos\theta}{\cos\theta+\sin\theta}=0 \Leftrightarrow 5\cos\theta=0 \Leftrightarrow \theta=k\dfrac{\pi}{2} \Rightarrow y=\dfrac{5\sin\theta}{\cos\theta+\sin\theta}=5,$ so the y-intercept is $(0,5)$. Similarly, the x-intercept is $(5,0)$. The distance between these points is $\sqrt{5^2+5^2}=\sqrt{50}=5\sqrt{2},$ option (**C**).

100. Note that $r=2-2\cos\theta \Rightarrow dr/d\theta=2\sin\theta$. The slope of the tangent line is

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\left(\frac{dr}{d\theta}\right)\sin\theta+r\cos\theta}{\left(\frac{dr}{d\theta}\right)\cos\theta-r\sin\theta}=\frac{2\sin\theta(\sin\theta)+(2-2\cos\theta)\cos\theta}{2\sin\theta(\cos\theta)-(2-2\cos\theta)\sin\theta}=\frac{\sin^2\theta+\cos\theta-\cos^2\theta}{2\cos\theta\sin\theta-\sin\theta}.$$

At the point $\left(2-\sqrt{2},\dfrac{7\pi}{4}\right)$ this slope is $\dfrac{\sin^2\left(\frac{7\pi}{4}\right)+\cos\left(\frac{7\pi}{4}\right)-\cos^2\left(\frac{7\pi}{4}\right)}{2\cos\left(\frac{7\pi}{4}\right)\sin\left(\frac{7\pi}{4}\right)-\sin\left(\frac{7\pi}{4}\right)}=\dfrac{\sqrt{2}}{\sqrt{2}-2}\approx-2.414,$ choice (**A**).

102. The distance between a point and the origin (the pole) on a polar curve is $|r|$, so we need to maximize $r^2=(2\theta\sin\theta-1)^2 \Leftrightarrow 2r\dfrac{dr}{d\theta}=2(2\theta\sin\theta-1)(2\theta\cos\theta+2\sin\theta)$

$\Rightarrow \dfrac{dr}{d\theta}=(2\theta\cos\theta+2\sin\theta)$. We find the critical points by solving $\dfrac{dr}{d\theta}=0 \Leftrightarrow$

$2\theta\cos\theta+2\sin\theta=0 \Leftrightarrow \theta=-\dfrac{\sin\theta}{\cos\theta}=\tan\theta \Leftrightarrow \theta=0,\ \theta=a\approx2.028757838,$ or $\theta=b\approx4.9131804$

When $\theta=0,\ |r|=1,$ and when $\theta=2\pi,\ |r|=1.$ When $\theta=a,\ |r|\approx2.639.$ and when $\theta=b,\ |r|=10.6$
So the distance is maximized when $\theta=b,$ and the polar coordinates of the point on this curve that farthest from the origin are $(-10.629,4.913)$.

103. The slope of the tangent line is $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=\dfrac{\left(\frac{dr}{d\theta}\right)\sin\theta+r\cos\theta}{\left(\frac{dr}{d\theta}\right)\cos\theta-r\sin\theta}=\dfrac{1\cdot\sin\theta+\theta\cdot\cos\theta}{1\cdot\cos\theta-\theta\cdot\sin\theta}.$ Whe

$\theta=\frac{\pi}{2}$, this slope is $\dfrac{\sin\left(\frac{\pi}{2}\right)+\frac{\pi}{2}\cdot\cos\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)-\frac{\pi}{2}\cdot\sin\left(\frac{\pi}{2}\right)}=\dfrac{1+0}{0-\frac{\pi}{2}\cdot(-1)}=\dfrac{2}{\pi},$ choice (**A**).