p. 811: 7-17 odd, 22, 23-27 odd, 29-30, 31-41 odd, 43-46, 49, 64, 70-73

7. $r = \cos\theta,\ 0 \le \theta \le \pi/6.$

$$A = \int_0^{\pi/6} \tfrac{1}{2} r^2 d\theta = \int_0^{\pi/6} \tfrac{1}{2}\cos^2\theta\, d\theta = \tfrac{1}{2}\int_0^{\pi/6} \tfrac{1}{2}(1+\cos 2\theta)\, d\theta = \tfrac{1}{4}\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_0^{\pi/6}$$
$$= \tfrac{1}{4}\left(\tfrac{\pi}{6} + \tfrac{1}{2}\cdot\tfrac{1}{2}\sqrt{3}\right) = \tfrac{\pi}{24} + \tfrac{1}{16}\sqrt{3}$$

9. $r = 1/\theta + \cos\theta,\ \pi/2 \le \theta \le 2\pi.$

$$A = \int_{\pi/2}^{2\pi} \tfrac{1}{2} r^2 d\theta = \int_{\pi/2}^{2\pi} \frac{1}{2}\left(\frac{1}{\theta}\right)^2 d\theta = \int_{\pi/2}^{2\pi} \frac{1}{2}\theta^{-2}\, d\theta = \frac{1}{2}\left[-\frac{1}{\theta}\right]_{\pi/2}^{2\pi}$$
$$= \tfrac{1}{2}\left(-\tfrac{1}{2\pi} + \tfrac{2}{\pi}\right) = \tfrac{1}{2}\left(-\tfrac{1}{2\pi} + \tfrac{4}{2\pi}\right) = \tfrac{3}{4\pi}$$

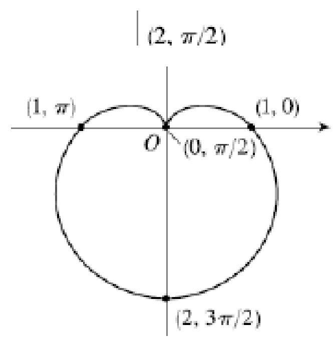
11. $r = 2 + \cos\theta,\ \pi/2 \le \theta \le \pi.$

$$A = \int_{\pi/2}^{\pi} \tfrac{1}{2} r^2 d\theta = \int_{\pi/2}^{\pi} \tfrac{1}{2}(2+\cos\theta)^2\, d\theta = \int_{\pi/2}^{\pi} \tfrac{1}{2}\left(4 + 4\cos\theta + \cos^2\theta\right) d\theta$$
$$= \int_{\pi/2}^{\pi} \tfrac{1}{2}\left[4 + 4\cos\theta + \tfrac{1}{2}(1+\cos 2\theta)\right] d\theta = \int_{\pi/2}^{\pi} \left(\tfrac{9}{4} + 2\cos\theta + \tfrac{1}{4}\cos 2\theta\right) d\theta \ = \left[\tfrac{9}{4}\theta + 2\sin\theta + \tfrac{1}{8}\sin 2\theta\right]_{\pi/2}^{\pi}$$
$$= \left(\tfrac{9\pi}{4} + 0 + 0\right) - \left(\tfrac{9\pi}{8} + 2 + 0\right) = \tfrac{9\pi}{8} - 2$$

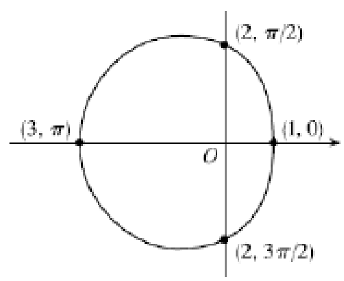
13. $r = \sqrt{\ln\theta},\ 1 \le \theta \le 2\pi.$

$$A = \int_1^{2\pi} \tfrac{1}{2}(\sqrt{\ln\theta})^2\, d\theta = \tfrac{1}{2}\int_1^{2\pi} \tfrac{1}{2}\ln\theta\, d\theta = \left[\tfrac{1}{2}\theta\ln\theta\right]_1^{2\pi} - \int_1^{2\pi} \tfrac{1}{2}\, d\theta \qquad \begin{bmatrix} u = \ln\theta & dv = \tfrac{1}{2}\, d\theta \\ du = (1/\theta)\, d\theta & v = \tfrac{1}{2}\theta \end{bmatrix}$$
$$= [\pi\ln(2\pi) - 0] - \left[\tfrac{1}{2}\theta\right]_1^{2\pi} = \pi\ln(2\pi) - \pi + \tfrac{1}{2}$$

15. $$A = \int_0^{2\pi} \tfrac{1}{2} r^2 d\theta = \int_0^{2\pi} \tfrac{1}{2}(1-\sin\theta)^2 d\theta$$
$$= \tfrac{1}{2}\int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)^2 d\theta = \tfrac{1}{2}\int_0^{2\pi} (1 - 2\sin\theta + \tfrac{1}{2}(1-\cos 2\theta))\, d\theta$$
$$= \tfrac{1}{2}\int_0^{2\pi} \left(\tfrac{3}{2} - 2\sin\theta - \tfrac{1}{2}\cos 2\theta\right) d\theta = \tfrac{1}{2}\left[\tfrac{3}{2}\theta + 2\cos\theta - \tfrac{1}{4}\sin 2\theta\right]_0^{2\pi}$$
$$= \tfrac{1}{2}\left[(3\pi + 2) - 2\right] = \tfrac{3\pi}{2}$$

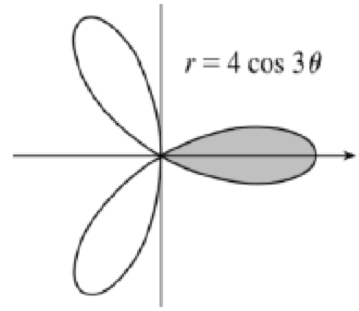


17. $$A = \int_0^{2\pi} \tfrac{1}{2} r^2 d\theta = \int_0^{2\pi} \tfrac{1}{2}(2-\cos\theta)^2 d\theta = \int_0^{2\pi} \tfrac{1}{2}(4 - 4\cos\theta + \cos^2\theta)\, d\theta$$
$$= \int_0^{2\pi} \tfrac{1}{2}\left[4 - 4\cos\theta + \tfrac{1}{2}(1+\cos 2\theta)\right] d\theta = \int_0^{2\pi} \left(\tfrac{9}{4} - 2\cos\theta + \tfrac{1}{4}\cos 2\theta\right) d\theta$$
$$= \left[\tfrac{9}{4}\theta - 2\sin\theta + \tfrac{1}{8}\sin 2\theta\right]_0^{2\pi} = \left(\tfrac{9\pi}{2} - 0 + 0\right) - (0 - 0 + 0) = \tfrac{9\pi}{2}$$



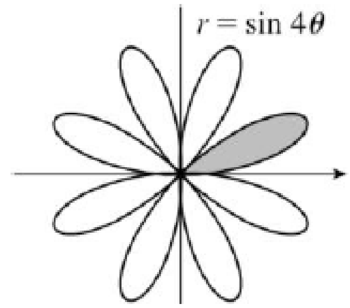
22. The area of this petal is $\int_0^{\pi/3} \tfrac{1}{2} r^2\, d\theta = \int_0^{\pi/3} 3\sin 3\theta\, d\theta = \left[-\cos 3\theta\right]_0^{\pi/3} = -\cos\pi - (-\cos 0) = 1 + 1 = 2,$ choice (**C**).

23. The curve passes through the pole when $r=0 \Rightarrow 4\cos 3\theta = 0 \Rightarrow \cos 3\theta = 0 \Rightarrow 3\theta = \frac{\pi}{2} + \pi n \Rightarrow \theta + \frac{\pi}{6} + \frac{\pi}{3}n$. The part of the shaded loop above the polar axis is traced out for $\theta = 0$ to $\theta = \pi/6$, so we'll use $-\pi/6$ and $\pi/6$ as our limits of integration.



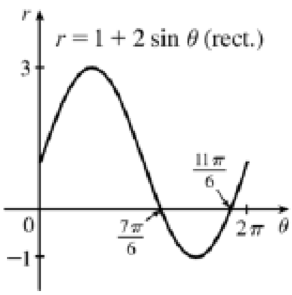
$$A = \int_{-\pi/6}^{\pi/6} \tfrac{1}{2}(4\cos 3\theta)^2\, d\theta = 2\int_0^{\pi/6} \tfrac{1}{2}(16\cos^2 3\theta)\, d\theta$$
$$= 16\int_0^{\pi/6} \tfrac{1}{2}(1+\cos 6\theta)\, d\theta = 8\left[\theta + \tfrac{1}{6}\sin 6\theta\right]_0^{\pi/6} = 8\left(\tfrac{\pi}{6}\right) = \tfrac{4}{3}\pi$$

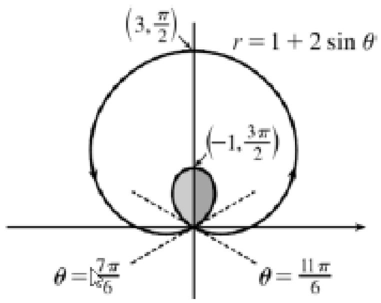
25. $r = 0 \Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = \pi n \Rightarrow \theta = \frac{\pi}{4}n$.



$$A = \int_0^{\pi/4} \tfrac{1}{2}(\sin 4\theta)^2\, d\theta = \tfrac{1}{2}\int_0^{\pi/4} \sin^2 4\theta\, d\theta = \int_0^{\pi/4} \tfrac{1}{2}(1-\cos 8\theta)\, d\theta$$
$$= \tfrac{1}{2}\left[\theta - \tfrac{1}{8}\sin 8\theta\right]_0^{\pi/4} = \tfrac{1}{4}\left(\tfrac{\pi}{4}\right) = \tfrac{1}{16}\pi$$

27.





This is a limacon, with inner loop traced out between $\theta = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$ [found by solving $r = 0$].

$$A = \int_{7\pi/6}^{3\pi/2} \tfrac{1}{2}(1+2\sin\theta)^2\, d\theta = \tfrac{1}{2}\int_{7\pi/6}^{3\pi/2}\left(1+4\sin\theta+4\sin^2\theta\right)d\theta = \tfrac{1}{2}\int_{7\pi/6}^{3\pi/2}[1+4\sin\theta+4\cdot\tfrac{1}{2}(1-\cos 2\theta)]\,d\theta$$
$$= \tfrac{1}{2}\left[\theta - 4\cos\theta + 2\theta - \sin 2\theta\right]_{7\pi/6}^{3\pi/2} = \left(\tfrac{9\pi}{4}\right) - \left(\tfrac{7\pi}{4} + \sqrt{3} - \tfrac{\sqrt{3}}{4}\right) = \tfrac{\pi}{2} - \tfrac{3\sqrt{3}}{4}$$

29. The curve is completely traced for $0 \le \theta \le \pi$. The area is

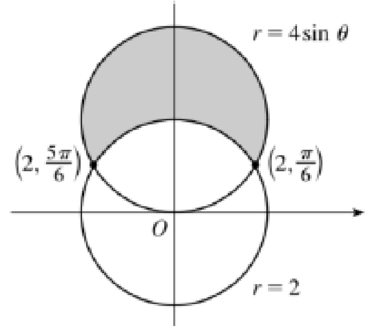
$$A = \int_0^{\pi} \tfrac{1}{2} r^2\, d\theta = \tfrac{1}{2}\int_0^{\pi}\left(\sqrt[3]{\sin\theta}\right)^2 d\theta = \tfrac{1}{2}\int_0^{\pi} \sin^{2/3}\theta\, d\theta,$$ which is choice (**A**).

30. The curve is a 3-petal rose and one complete petal is traced out for $\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$. So the total area bounded by the graph is

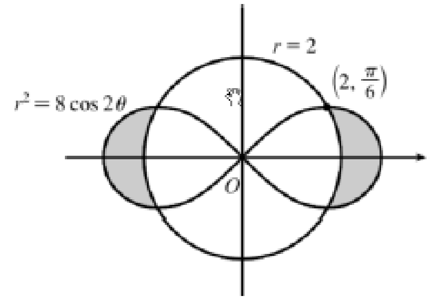
$$3\int_{\pi/2}^{5\pi/6} \tfrac{1}{2}(2\cos 3\theta)^2\, d\theta = \int_{\pi/2}^{5\pi/6} 6\cos^2 3\theta\, d\theta = \left[3\theta + \cos 3\theta \sin 3\theta\right]_{\pi/2}^{5\pi/6} = \left(\tfrac{5\pi}{2} - 0\right) - \left(\tfrac{3\pi}{2} - 0\right) = \pi,$$ (**C**).

31. $4\sin\theta = 2 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6} \Rightarrow$

$$A = \int_{\pi/6}^{5\pi/6} \tfrac{1}{2}[(4\sin\theta)^2 - 2^2]\,d\theta = \int_{\pi/6}^{5\pi/6}(8\sin^2\theta - 2)\,d\theta$$
$$= 2\int_{\pi/6}^{\pi/2}(8\cdot\tfrac{1}{2}(1-\cos 2\theta) - 2)\,d\theta = \int_{\pi/6}^{\pi/2}(4 - 8\cos 2\theta)\,d\theta$$
$$= \left[4\theta - 4\sin 2\theta\right]_{\pi/6}^{\pi/2} = (2\pi - 0) - \left(\tfrac{2\pi}{3} - 2\sqrt{3}\right) = \tfrac{4\pi}{3} + 2\sqrt{3}$$

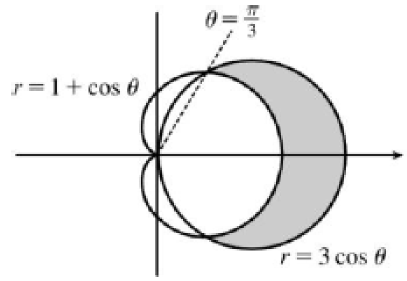


33. To find the area inside the lemniscate $r^2 = 8\cos 2\theta$ and outside the circle $r = 2$, we first note that the two curves intersect when $r^2 = 8\cos 2\theta$ and $r = 2$, that is, when $\cos 2\theta = \frac{1}{2}$. For $-\pi \le \theta \le \pi$, $\cos 2\theta = \frac{1}{2} \Leftrightarrow 2\theta = \pm\pi/3$ or $\pm\pi/3 \Leftrightarrow \theta = \pm\pi/6$ or $\pm 5\pi/6$. The figure shows that the desired area is 4 times the area between the curves from 0 to $\pi/6$. Thus,



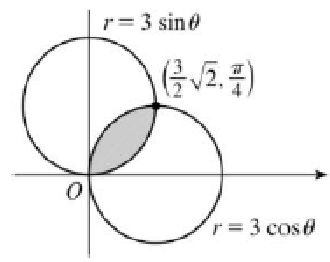
$$A = 4\int_0^{\pi 6}\left[\tfrac{1}{2}(8\cos 2\theta) - \tfrac{1}{2}(2)^2\right]d\theta = 8\int_0^{\pi/6}(2\cos 2\theta - 1)\,d\theta$$
$$= 8\left[\sin 2\theta - \theta\right]_0^{\pi/6} = 8\left(\sqrt{3}/2 - \pi/6\right) = 4\sqrt{3} - 4\pi/3$$

35. $3\cos\theta = 1 + \cos\theta \Leftrightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ or $-\frac{\pi}{3}$.



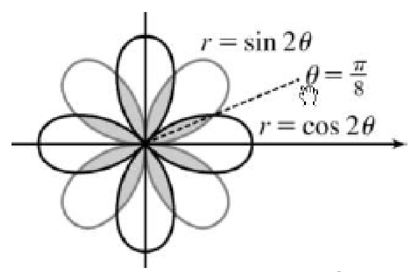
$$A = 2\int_0^{\pi/3}\tfrac{1}{2}\left[(3\cos\theta)^2 - (1+\cos\theta)^2\right]d\theta$$
$$= \int_0^{\pi/3}(8\cos^2\theta - 2\cos\theta - 1)\,d\theta = \int_0^{\pi/3}\left[4(1+\cos 2\theta) - 2\cos\theta - 1\right]d\theta$$
$$= \int_0^{\pi/3}(3 + 4\cos 2\theta - 2\cos\theta)\,d\theta = \left[3\theta + 2\sin 2\theta - 2\sin\theta\right]_0^{\pi/3}$$
$$= \pi + \sqrt{3} - \sqrt{3} = \tfrac{\pi}{2}$$

37. $3\sin\theta = 3\cos\theta \Rightarrow \dfrac{3\sin\theta}{3\cos\theta} = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4} \Rightarrow$



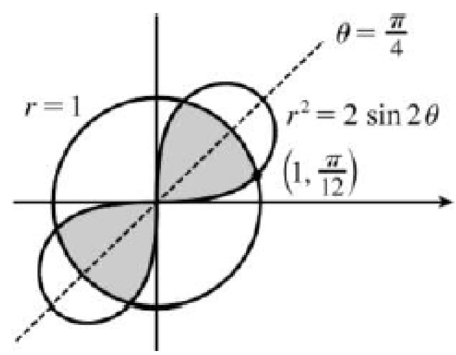
$$A = 2\int_0^{\pi/4}\tfrac{1}{2}(3\sin\theta)^2\,d\theta = \int_0^{\pi/4}9\sin^2\theta\,d\theta = \int 9\cdot\tfrac{1}{2}(1-\cos 2\theta)\,d\theta$$
$$= \int_0^{\pi/4}\left(\tfrac{9}{2} - \tfrac{9}{2}\cos 2\theta\right)d\theta = \left[\tfrac{9}{2}\theta - \tfrac{9}{4}\sin 2\theta\right]_0^{\pi/4} = \left(\tfrac{9\pi}{8} - \tfrac{9}{4}\right) - (0-0)$$
$$= \tfrac{9\pi}{8} - \tfrac{9}{4}$$

39. $\sin 2\theta = \cos 2\theta \Rightarrow \tan 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{8} \Rightarrow$



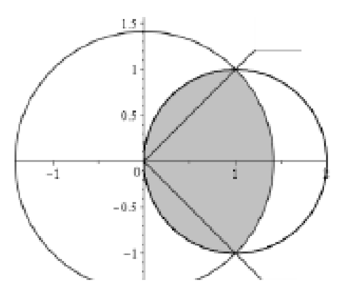
$$A = 8\cdot 2\int_0^{\pi/8}\tfrac{1}{2}\sin^2 2\theta\,d\theta = 8\int_0^{\pi/8}\tfrac{1}{2}(1-\cos 4\theta)\,d\theta$$
$$= 4\left[\theta - \tfrac{1}{2}\sin 4\theta\right]_0^{\pi/8} = 4\left(\tfrac{\pi}{8} - \tfrac{1}{4}\cdot 1\right) = \tfrac{\pi}{2} - 1$$

41. From the figure, we see that the shaded region is 4 times the shaded region from $\theta = 0$ to $\theta = \pi/4$. $r^2 = 2\sin^2\theta$ and $r = 1 \Rightarrow$ $2\sin 2\theta = 1^2 \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{12}$.



$$A = 4\int_0^{\pi/12}\tfrac{1}{2}(2\sin 2\theta)\,d\theta + 4\int_{\pi/12}^{\pi/4}\tfrac{1}{2}(1)^2\,d\theta$$
$$= \int_0^{\pi/12}4\sin 2\theta\,d\theta + \int_{\pi/12}^{\pi/4}2\,d\theta = \left[-2\cos\theta\right]_0^{\pi/12} + \left[2\theta\right]_{\pi/12}^{\pi/4}$$
$$= \left(-\sqrt{3} + 2\right) + \left(\tfrac{\pi}{2} - \tfrac{\pi}{6}\right) = -\sqrt{3} + 2 + \tfrac{\pi}{3}$$

43. $2\cos\theta = \sqrt{2} \Leftrightarrow \cos\theta = \frac{\sqrt{2}}{2} \Leftrightarrow$ the curves intersect when $\theta = \frac{\pi}{4}$, or $\frac{7\pi}{4}$. By symmetry, the area bounded by the curves is



$$A = 2\left[\int_0^{\pi/4}\tfrac{1}{2}\left(\sqrt{2}\right)^2 d\theta + \int_{\pi/4}^{\pi/2}\tfrac{1}{2}(2\cos\theta)^2 d\theta + \right]$$
$$= \int_0^{\pi/4}2\,d\theta + \int_{\pi/4}^{\pi/2}4\cos^2\theta\,d\theta = \pi - 1 \approx 2.142, \text{choice } \mathbf{(C)}.$$

44. The area in the cardioid and outside the circle of radius 1 is
$A=\int_0^{\pi}\frac{1}{2}(1+\sin\theta)^2\,d\theta-\int_0^{\pi/2}1^2\,d\theta=2+\frac{\pi}{4}\approx 2.785$, choice (**C**).

45. $A=2\left[\frac{1}{4}\pi\cdot 4^2+\int_{\pi/2}^{\pi}\frac{1}{2}(4+4\cos\theta)^2\,d\theta\right]=8\pi+\int_{\pi/2}^{\pi}(4+4\cos\theta)^2\,d\theta=20\pi-32\approx 30.832$, choice (**A**).

46. The curves intersect at $\theta=\frac{2\pi}{3}$. The area is $A=2\left[\int_{\pi/2}^{2\pi/3}\frac{1}{2}(-6\cos\theta)^2\,d\theta+\int_{2\pi/3}^{\pi}\frac{1}{2}(2-2\cos\theta)^2\,d\theta\right]$
$=\int_{\pi/2}^{2\pi/3}36\cos^2\theta\,d\theta+\int_{2\pi/3}^{\pi}(4-8\cos\theta+4\cos^2\theta)\,d\theta=5\pi$, option (**D**).

49. The area of the region inside the graph of $r=1+\sin\theta$ and outside the graph of $r=1$ is
$\frac{1}{2}\int_0^{\pi}(1+\sin\theta)^2\,d\theta-\frac{1}{2}\int_0^{\pi}1^2\,d\theta=\frac{1}{2}\int_0^{\pi}(1+\sin\theta)^2-1\,d\theta=\frac{1}{2}\int_0^{\pi}(1+2\sin\theta+\sin^2\theta-1)\,d\theta$
$=\frac{1}{2}\int_0^{\pi}(2\sin\theta+\sin^2\theta)\,d\theta=\frac{1}{2}\int_0^{\pi}\sin\theta(2+\sin\theta)\,d\theta$, (**A**).

64. The inner loop of the limacon is traced with $\frac{2\pi}{3}\le\theta\le\frac{4\pi}{3}$, so the area of this loop is
$A=\int_{2\pi/3}^{4\pi/3}\frac{1}{2}(1+2\cos\theta)^2\,d\theta=\pi-\frac{3\sqrt{3}}{2}\approx 0.544$, option (**C**).

70. (a) The area of the region R is the area of the circle of radius 3, below the x-axis $\left(\frac{1}{2}\pi\cdot 3^2\right)$ plus the area under twice the area under this circle from $\theta=0$ to $\theta=\pi/6$, plus the area under the curve $r=4-2\sin\theta$ from $\theta=\pi/6$ to $\theta=5\pi/6$. So,
$A=\dfrac{9\pi}{2}+2\cdot\int_0^{\pi/6}\frac{1}{2}\cdot 3^2\,d\theta+2\cdot\int_{\pi/6}^{5\pi/6}\frac{1}{2}\cdot(4-2\sin^2\theta)\,d\theta=\dfrac{9\pi}{2}+\int_0^{\pi/6}3^2\,d\theta+\int_{\pi/6}^{5\pi/6}(4-2\sin^2\theta)\,d\theta$
$=18\pi-\dfrac{15\sqrt{3}}{2}.$

(b) $r=4-2\sin\theta,\ \theta=t^2.\ 1=\dfrac{dr}{dt}=\dfrac{dr}{d\theta}\cdot\dfrac{d\theta}{dt}\Leftrightarrow 1=-2\cos\theta\cdot 2t=-4t\cos t^2\Leftrightarrow t\overset{\text{CAS}}{\approx}1.327.$

(c) $x(t)=r\cos t=\left(4-2\sin t^2\right)\cos t.\ x(t)=-1\Leftrightarrow -1=\left(4-2\sin t^2\right)\cos t\Leftrightarrow t\approx 1.808.$
$y(t)=r\cdot\sin t=(4-2\sin t^2)\sin t\Rightarrow y'(t)=(4-2\sin t^2)\cos t+\sin t\left(-4t\cos t^2\right)$
$x'(t)=-\left(4-2\sin t^2\right)\sin t+\cos t(-4t\cos t^2)$. $\mathbf{v}(1.808)=\langle x'(1.808),y'(1.808)\rangle\approx\langle -5.821,5.973\rangle$

71. (a) The distance between any point and the origin is $|r|$. We can maximize this distance by finding the critical points of r^2, that is $2r\dfrac{dr}{d\theta}=2r\left(\cos\theta-\theta\sin\theta\right)=2\left(\theta\cos\theta\right)\left(\cos\theta-\theta\sin\theta\right)$.
$\dfrac{dr}{d\theta}=0\Leftrightarrow\cos\theta-\theta\sin\theta=0\Leftrightarrow\cos\theta=\theta\sin\theta\Leftrightarrow\csc\theta=\theta\Leftrightarrow\theta=a\approx 0.8603335890$, or
$\theta=b\approx 3.425618459$. Now we evaluate $|r|$ at each of these critical points and the endpoints to determine the maximum distance from the origin:
$r(0)=0,\ r\left(\frac{3\pi}{2}\right)=0,\ |r(a)|\approx 0.561$, and $|r(b)|\approx|-3.288|=3.288$. Therefore, the maximum distance is approximately 3.288 and it occurs when $\theta\approx 3.426$.
(b) The curve is completely traced for $0\le\theta\le\frac{3\pi}{2}$, and the inner loop is traced when $0\le\theta\le\frac{\pi}{2}$.
Therefore, the area between the loops is $A=\frac{1}{2}\int_{\pi/2}^{3\pi/2}(\theta\cos\theta)^2\,d\theta=\frac{13}{48}\pi^3-\frac{1}{8}\pi\approx 8.005$.

(c) $\dfrac{-2}{\pi-2}=\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}\Rightarrow\dfrac{dx}{d\theta}\left(\dfrac{-2}{\pi-2}\right)=\dfrac{dy}{d\theta}=\dfrac{1}{2}\Rightarrow\dfrac{dx}{d\theta}=\dfrac{1}{2}\cdot\dfrac{\pi-2}{-2}=\dfrac{2-\pi}{4}$

(d) $\dfrac{dr}{dy}=\dfrac{dr}{d\theta}\cdot\dfrac{d\theta}{dy}=\left(\cos\theta-\theta\sin\theta\right)\cdot 2=2\cdot\dfrac{dr}{d\theta}$

72. (a) $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=\dfrac{r'(\theta)\sin\theta+r(\theta)\cos\theta}{r'(\theta)\cos\theta-r(\theta)\cos\theta}=\dfrac{(3+\sin\theta)\sin\theta+(3\theta-\cos\theta)\cos\theta}{(3+\sin\theta)\cos\theta-(3\theta-\cos\theta)\cos\theta}$

When $\theta=\dfrac{\pi}{6}$, $\dfrac{dy}{dx}=\dfrac{\left(3+\sin\left(\frac{\pi}{6}\right)\right)\sin\theta+\left(\frac{\pi}{2}-\cos\left(\frac{\pi}{6}\right)\right)\cos\frac{\pi}{6}}{\left(3+\sin\left(\frac{\pi}{6}\right)\right)\cos\theta-\left(\frac{\pi}{2}-\cos\left(\frac{\pi}{6}\right)\right)\cos\frac{\pi}{6}}=-\dfrac{\sqrt{3}\left(\sqrt{3}\pi+4\right)}{3\pi-21-3\sqrt{3}}\approx 0.975$

(b) $A=\int_{\pi/2}^{\pi}\frac{1}{2}r^2\,d\theta=\int_{\pi/2}^{\pi}\frac{1}{2}(3\theta+\cos\theta)^2\,d\theta=-3+\frac{21}{16}\pi^3-\frac{11}{8}\pi\approx 33.376$

(c) $y=1=(3\theta+\cos\theta)\sin\theta\Rightarrow\theta\overset{\text{CAS}}{\approx}3.017$. At this point, $x=(3\theta+\cos\theta)\cos\theta\approx-7.997$.

(d) $x=r\cos\theta=\left(3\theta+\cos\theta\right)\cos\theta$.

$\dfrac{dx}{dt}=\dfrac{dx}{d\theta}\cdot\dfrac{d\theta}{dt}=\left[(3\theta+\cos\theta)(-\sin\theta)+\cos\theta\cdot(3-\sin\theta)\right]\cdot 2$

When $\theta=\frac{5\pi}{6}$, $\dfrac{dx}{dt}=2\left[\left(3\left(\frac{5\pi}{6}\right)+\cos\frac{5\pi}{6}\right)\left(-\sin\frac{5\pi}{6}\right)+\cos\frac{5\pi}{6}\cdot\left(3-\sin\frac{5\pi}{6}\right)\right]=-\frac{5\pi}{2}-2\sqrt{3}\approx-11.381$.

When $\theta=\frac{5\pi}{6}$, the x-coordinate of the particle is decreasing at a rate of 11.381.

73. (a) The area of the region in the first quadrant bounded by the curve $y=2-4\sin\theta$ and the polar axis is $A=\frac{1}{2}\int_0^{\pi/6}(2-4\sin\theta)^2\,d\theta$.

(b) $x=r(\theta)\cos\theta=(2-4\sin\theta)\cdot\cos\theta$ and $y=r(\theta)\sin(\theta)=(2-4\sin\theta)\cdot\sin\theta$.

$\dfrac{dx}{dt}=(4-2\sin\theta)(-\sin\theta)\dfrac{d\theta}{dt}+\cos\theta(-2\cos\theta)\dfrac{d\theta}{dt}=\left(-4\sin\theta+2\sin^2\theta-2\cos^2\theta\right)\dfrac{d\theta}{dt}$

$\quad=\left(2-4\sin\theta-4\cos^2\theta\right)d\theta/dt$

$\dfrac{dy}{dt}=(4-2\sin\theta)(\cos\theta)\dfrac{d\theta}{dt}+\sin\theta(-2\cos\theta)\dfrac{d\theta}{dt}=\left(4\cos\theta-2\cos\theta\sin\theta-2\cos\theta\sin\theta\right)\dfrac{d\theta}{dt}$

$\quad=\left(4\cos\theta-4\cos\theta\sin\theta\right)\dfrac{d\theta}{dt}$

(c) The slope of the tangent line is

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\frac{dr}{d\theta}\cdot\sin\theta+r\cdot\cos\theta}{\frac{dr}{d\theta}\cdot\cos\theta-r\cdot\sin\theta}=\frac{(-4\cos\theta)\sin\theta+(2-4\sin\theta)\cos\theta}{(-4\cos\theta)\cos\theta-(2-4\sin\theta)\sin\theta}$$

$$=\frac{-8\cos\theta\sin\theta+2\cos\theta}{-4\cos^2\theta-2\sin\theta+4\sin^2\theta}.$$

When $\theta=\frac{\pi}{3}$, the slope is $\dfrac{2\sqrt{3}-1}{\sqrt{3}-2}\approx-9.196$. When $\theta=\frac{\pi}{3}$, $x=\left(2-4\sin\frac{\pi}{3}\right)\cos\frac{\pi}{3}=1-\sqrt{3}\approx-0.732$,

and $y=\left(2-4\sin\frac{\pi}{3}\right)\sin\frac{\pi}{3}=\dfrac{\left(2-2\sqrt{3}\right)\sqrt{3}}{2}\approx 1.268$. Thus the equation of the tangent line is approximately $y-1.268=-9.196(x+0.732)$.