p. 487: 7-8, 15-16, 21-31 odd, 32, 40-44, 61, 73

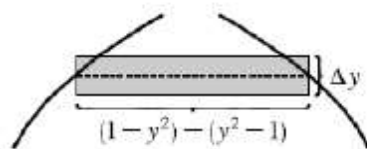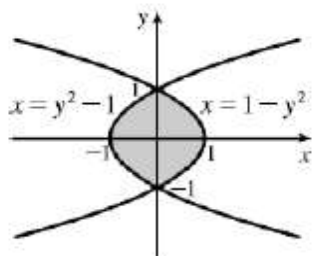7. $A = \int_{y=-1}^{y=1} (x_R - x_L)\,dy = \int_{-1}^{1}\left[e^y - (y^2 - 2)\right]dy = \int_{-1}^{1}(e^y - y^2 + 2)\,dy$

$= \left[e^y - \tfrac{1}{3}y^3 + 2y\right]_{-1}^{1} = (e^1 - \tfrac{1}{3} + 2) - (e^{-1} + \tfrac{1}{3} - 2) = e - \dfrac{1}{e} + \dfrac{10}{3}$

8. $A = \int_{0}^{3}\left[(2y - y^2) - (y^2 - 4y)\right]dy = \int_{0}^{3}(-2y^2 + 6y)\,dy = \left[-\tfrac{2}{3}y^3 + 3y^2\right]_{0}^{3} = (-18 + 27) - 0 = 9$

15. The curves intersect when $1 - y^2 = y^2 - 1 \Leftrightarrow 2 = 2y^2 \Leftrightarrow y^2 = 1 \Leftrightarrow y = \pm 1$.
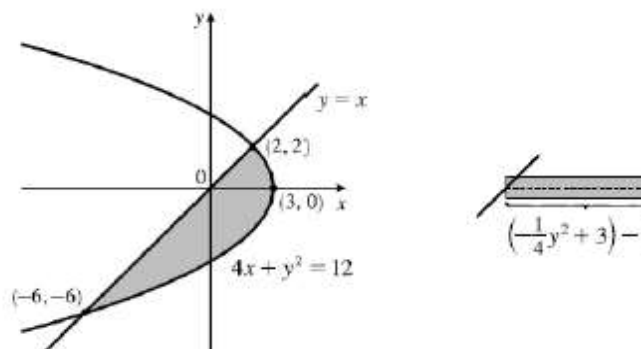
$A = \int_{-1}^{1}\left[(1 - y^2) - (y^2 - 1)\right]dy = \int_{-1}^{1} 2(1 - y^2)\,dy = 2\cdot 2\int_{0}^{1}(1 - y^2)\,dy = 4\left[y - \tfrac{1}{3}y^3\right]_{0}^{1} = 4(1 - \tfrac{1}{3}) = \tfrac{8}{3}$

16. $4x + x^2 = 12 \Leftrightarrow (x + 6)(x - 2) = 0 \Leftrightarrow$
$x = -6 \text{ or } x = 2, \text{ so } y = -6 \text{ or } y = 2, \text{ and }$

$A = \int_{-6}^{2}\left[(-\tfrac{1}{4}y^2 + 3) - y\right]dy$

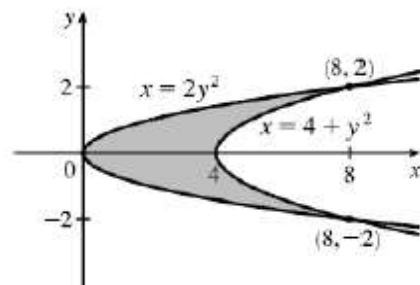$= \left[-\tfrac{1}{12}y^3 - \tfrac{1}{2}y^2 + 3y\right]_{-6}^{2}$

$= (-\tfrac{2}{3} - 2 + 6) - (18 - 18 - 18) = 22 - \tfrac{2}{3} = \tfrac{64}{3}$

21. $2y^2 = 4 + y^2 \Leftrightarrow y^2 = 4 \Leftrightarrow y = \pm 2$, so

$A = \int_{-2}^{2}\left[(4 + y^2) - 2y^2\right]dy$
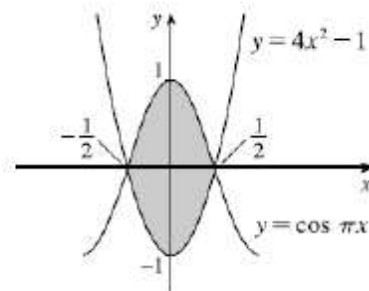
$= 2\int_{0}^{2}(4 - y^2)\,dy$ [by symmetry]

$= 2\left[4y - \tfrac{1}{3}y^3\right]_{0}^{2} = 2(8 - \tfrac{8}{3}) = \tfrac{32}{3}$

23. By inspection, the curves intersect at $x = \pm\tfrac{1}{2}$.

$A = \int_{-1/2}^{1/2}\left[\cos \pi x - (4x^2 - 1)\right]dx$

$= 2\int_{0}^{1/2}\left(\cos \pi x - 4x^2 + 1\right)dx$ [by symmetry]

$= 2\left[\tfrac{1}{\pi}\sin \pi x - \tfrac{4}{3}x^3 + x\right]_{0}^{1/2} = 2\left[(\tfrac{1}{\pi} - \tfrac{1}{6} + \tfrac{1}{2}) - 0\right] = 2(\tfrac{1}{\pi} + \tfrac{1}{3}) = \tfrac{2}{\pi} + \tfrac{2}{3}$
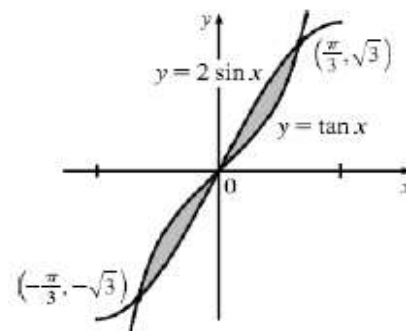
25. The curves intersect when $\tan x = 2\sin x$ (on $\left[-\pi/3, \pi/3\right]$) $\Leftrightarrow$

$\sin x = 2\sin x \cos x \Leftrightarrow 2\sin x \cos x - \sin x = 0 \Leftrightarrow$

$\sin x(2\cos x - 1) = 0 \Leftrightarrow \sin x = 0$ or $\cos x = \frac{1}{2} \Leftrightarrow x = 0$ or $x = \pm\frac{\pi}{3}$.

$A = 2\int_0^{\pi/3}(2\sin x - \tan x)\,dx$ [by symmetry]

$\quad = 2\left[-2\cos x - \ln|\sec x|\right]_0^{\pi/3} = 2\left[(-1-\ln 2)-(-2-0)\right]$

$\quad = 2(1-\ln 2) = 2 - 2\ln 2$

27. The curves intersect when

$\sqrt[3]{2x} = \frac{1}{8}x^2 \Leftrightarrow 2x = \frac{1}{(2^3)^3}x^6 \Leftrightarrow 2^{10}x = x^6 \Leftrightarrow x^6 - 2^{10}x = 0 \Leftrightarrow$
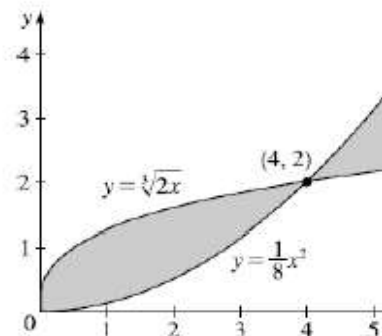
$x\left(x^5 - 2^{10}\right) = 0 \Leftrightarrow x = 0$ or $x^5 = 2^{10}$ $x = 0$ or $x = 2^2 = 4$, so for

$0 \le x \le 6$,

$A = \int_0^4\left(\sqrt[3]{2x} - \frac{1}{8}x^2\right)dx + \int_4^5\left(\frac{1}{8}x^2 - \sqrt[3]{2x}\right)dx$

$\quad = \left[\frac{3}{4}\sqrt[3]{2}x^{4/3} - \frac{1}{24}x^3\right]_0^4 + \left[\frac{1}{24}x^3 - \frac{3}{4}\sqrt[3]{2}x^{4/3}\right]_4^6$

$\quad = \left(\frac{3}{4}\sqrt[3]{2}\cdot 4\sqrt[3]{4} - \frac{64}{24}\right) - (0-0) + \left(\frac{216}{24} - \frac{3}{4}\sqrt[3]{2}\cdot 6\sqrt[3]{6}\right) - \left(\frac{64}{24} - \frac{3}{4}\sqrt[3]{2}\cdot 4\sqrt[3]{4}\right)$
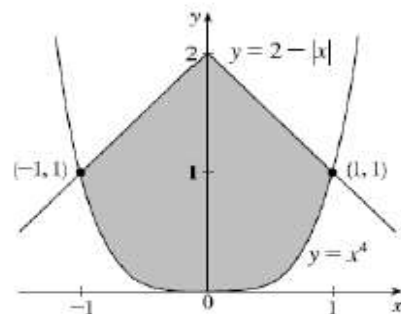
$\quad = 6 - \frac{8}{3} + 9 - \frac{9}{2}\sqrt[3]{12} - \frac{8}{3} + 6 = \frac{47}{3} - \frac{9}{2}\sqrt[3]{12}$

29. By inspection, we see that the curves intersect at $x = \pm 1$ and that the area of the region enclosed by the curves is twice the area enclosed in the first quadrant.

$A = 2\int_0^1\left[(2-x)-x^4\right]dx = 2\left[2x - \frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1$

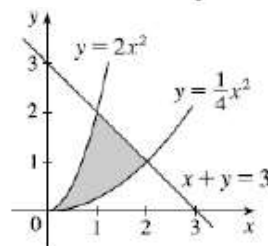$\quad = 2\left[\left(2-\frac{1}{2}-\frac{1}{5}\right)-0\right] = 2\left(\frac{13}{10}\right) = \frac{13}{5}$

31. $\frac{1}{4}x^2 = -x+3 \Leftrightarrow x^2 + 4x - 12 = 0 \Leftrightarrow (x+6)(x-2) = 0 \Leftrightarrow x = -6$ or 2 and

$2x^2 = -x+3 \Leftrightarrow 2x^2 + x - 3 = 0 \Leftrightarrow (2x+3)(x-1) = 0 \Leftrightarrow x = -\frac{3}{2}$ or 1,

for $x \ge 1$.

$A = \int_0^1\left(2x^2 - \frac{1}{4}x^2\right)dx + \int_1^2\left[(-x+3)-\frac{1}{4}x^2\right]dx = \int_0^1\frac{7}{4}x^2\,dx + \int_1^2\left[-\frac{1}{4}x^2 - x + 3\right]dx$

$\quad = \frac{7}{12}x^3\Big]_0^1 + \left[-\frac{1}{12}x^3 - \frac{1}{2}x^2 + 3x\right]_1^2 = \frac{7}{12} + \left(-\frac{2}{3}-2+6\right) - \left(-\frac{1}{12}-\frac{1}{2}+3\right) = \frac{3}{2}$
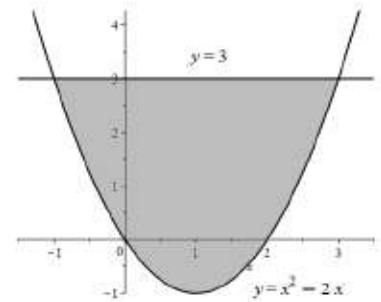
32. (a) Total area $= 12 + 27 = 39$.

(b) $f(x) \le g(x)$ for $0 \le x \le 2$ and $f(x) \ge g(x)$ for $2 \le x \le 5$, so

$\int_0^5[f(x)-g(x)]\,dx = \int_0^2[f(x)-g(x)]\,dx + \int_2^5[f(x)-g(x)]\,dx = -\int_0^2[g(x)-f(x)]\,dx + \int_2^5[f(x)-g(x)]\,dx$
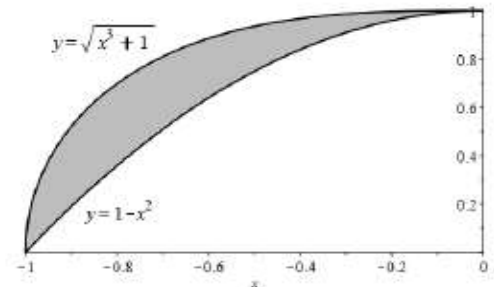
$\quad = -(12) + 27 = 15$.

40. The curves intersect at $x=-1$ and $x=3$, so the area of the region bound by the curves is

$$A=\int_{-1}^{3}\left[3-(x^2-2x)\right]dx$$

$$=\left[3x-\tfrac{1}{3}x^3+x^2\right]_{-1}^{3}$$

$$=(9-9+9)-(-3+\tfrac{1}{3}+1)=\tfrac{32}{3},\text{ option (B)}.$$

41. The curves intersect at $x=-1$ and $x=0$, so the area of the region bound by these curves is

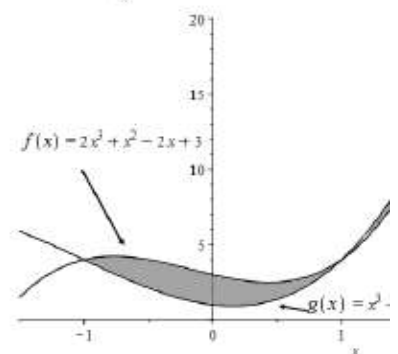$$A=\int_{-1}^{0}\left[\sqrt{x^3+1}-(1-x^2)\right]dx=0.1746,\text{ option (A)}.$$


42. Solving, we find that the curves intersect at $x=-1, x=1$ and $x=2$. The area of the region bounded by these curves is

$$A=\int_{-1}^{1}\left[f(x)-g(x)\right]dx+\int_{1}^{2}\left[g(x)-g(x)\right]dx$$

$$=\int_{-1}^{1}(x^3-2x^2-x+2)\,dx+\int_{1}^{2}(-x^3+2x^2+x-2)dx$$

$$=\left[\tfrac{1}{4}x^4-\tfrac{2}{3}x^3-\tfrac{1}{2}x^2+2x\right]_{-1}^{1}+\left[-\tfrac{1}{4}x^4+\tfrac{2}{3}x^3+\tfrac{1}{2}x^2-2x\right]_{1}^{2}$$

$$=\left(\tfrac{1}{4}-\tfrac{2}{3}-\tfrac{1}{2}+2\right)-\left(\tfrac{1}{4}+\tfrac{2}{3}-\tfrac{1}{2}-2\right)+\left(-4+\tfrac{16}{3}+2-4\right)-\left(-\tfrac{1}{4}+\tfrac{2}{3}+\tfrac{1}{2}-2\right)=\tfrac{37}{12}\approx3.084,\text{ option (C)}.$$

43. Using technology, we find that the curves intersect at $x=a\approx2.92627$, so the area bounded by the axis and these curves is $A=\int_{1}^{a}\ln x\,dx+\int_{a}^{4}(4-x)\,dx\approx1.792,$ option (B).
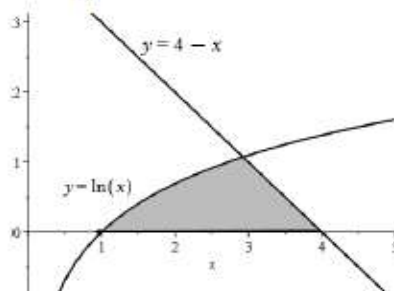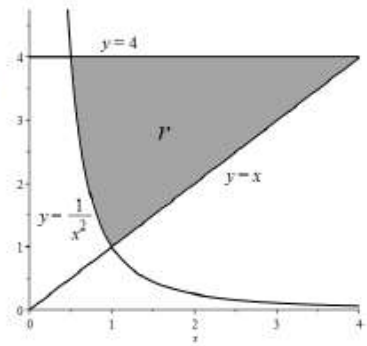
44. From the graph, we see that the curves $y=\dfrac{1}{x^2}$ and $y=4$ intersect at

$\left(\frac{1}{2},4\right)$, the curves $y=\dfrac{1}{x^2}$ and $y=x$ intersect at the point $(1,1)$, and the

curves $y=x$ and $y=4$ intersect at the point $(4,4)$. Thus, the area of

the region bounded by these curves is

$$A=\int_{1/2}^{1}\left(4-\frac{1}{x^2}\right)dx+\int_{1}^{4}(4-x)\,dx=\left[4x+\frac{1}{x}\right]_{1/2}^{1}+\left[4x-\tfrac{1}{2}x^2\right]_{1}^{4}$$

$$=(4+1)-(2+2)+(16-8)-(4-\tfrac{1}{2})=\tfrac{11}{2}$$

61. (a) At time $t=2$ seconds, Adam's velocity is $v_A(2)=\dfrac{40(2)}{\sqrt{2^2+20}}=\dfrac{80}{\sqrt{24}}\approx 16.2399$ ft/s, and Bassam's

velocity is $v_B(2)=\dfrac{36(2)}{\sqrt{2^2+10}}=\dfrac{72}{\sqrt{14}}\approx 19.243$ ft/s, so Bassam is traveling approximately 2.913 ft/s

faster than Adam.

(b) The bicyclists have equal velocity when $v_A(t)=v_B(t)\Leftrightarrow \dfrac{40t}{\sqrt{t^2+20}}=\dfrac{36t}{\sqrt{t^2+10}}\Leftrightarrow$

$\dfrac{\sqrt{t^2+10}}{\sqrt{t^2+20}}=\dfrac{36}{40}=0.9\Leftrightarrow\dfrac{t^2+10}{t^2+20}=0.81\Leftrightarrow t^2+10=0.81(t^2+20)\Leftrightarrow 0.19t^2=6.2\Leftrightarrow t^2=\tfrac{620}{19}\Leftrightarrow$

$t=\sqrt{\tfrac{620}{19}}\approx 5.712$ seconds $[t>0]$.

(c) We know that the area under the graph of $v_A(t)$ between $t=0$ and $t=20$ is $\int_{0}^{20}v_A(t)\,dt=s_A(20)$,

gives Adam's displacement after 20 seconds. Similarly, the area under the graph of $v_B(t)$ between

$t=0$ and $t=20$ is $\int_{0}^{20}v_B(t)\,dt=s_B(20)$ gives Bassam's displacement after 20 seconds. Then

$$s_A(20)=\int_{0}^{20}\left(\frac{40t}{\sqrt{t^2+20}}\right)dt=\left[40(t^2+20)^{1/2}\right]_{0}^{20}=40\left(\sqrt{420}-\sqrt{20}\right)\approx 640.871\text{ ft, and}$$

$$s_b(20)=\int_{0}^{20}\left(\frac{36t}{\sqrt{t^2+10}}\right)dt=\left[30(t^2+10)^{1/2}\right]_{0}^{20}=36\left(\sqrt{410}-\sqrt{10}\right)\approx 615.102\text{ ft. Thus, Adam is}$$

roughly 26 feet ahead of Bassam.

(d) $d_A(t)=\int_{0}^{t}\left(\dfrac{40x}{\sqrt{x^2+20}}\right)dx=\left[40(x^2+20)^{1/2}\right]_{0}^{t}=40\sqrt{t^2+20}-40\sqrt{20},$ and

$$d_B(t)=\int_{0}^{t}\left(\frac{36x}{\sqrt{x^2+10}}\right)dx=\left[36(x^2+10)^{1/2}\right]_{0}^{t}=36\sqrt{t^2+10}-36\sqrt{20}.$$

73. We first assume that $c > 0$, since c can be replaced by $-c$ in both equations without changing the graphs, and if $c = 0$ the curves do not enclose a region. We see from the graph that the enclosed area A lies between $x = -c$ and $x = c$, and by symmetry, it is equal to four times the area in the first quadrant. The enclosed area is

$$A = 4\int_0^c \left(c^2 - x^2\right)dx = 4\left[c^2 x - \tfrac{1}{3}x^3\right]_0^c = 4\left(c^3 - \tfrac{1}{3}c^3\right) = 4\left(\tfrac{2}{3}c^3\right) = \tfrac{8}{3}c^3.$$

So $A = 576 \Leftrightarrow \tfrac{8}{3}c^3 = 576 \Leftrightarrow c^3 = 216 \Leftrightarrow c = \sqrt[3]{216} = 6.$

Note that is $c = -6$ is another solution, since the graphs are the same.