p. 560: 5-41 EOO, 54, 57, 63-64

The symbols $\overset{s}{=}$ and $\overset{c}{=}$ indicate the use of substitutions $\{u = \sin x, du = \cos x\,dx\}$ and $\{u = \cos x, du = -\sin x\,dx\}$, respectively.

5. $\int \sin^2 x \cos^3 x\,dx = \int \sin^2 x \cos^2 x \cos x\,dx = \int \sin^2 x\left(1-\sin^2 x\right)\cos x\,dx$

$$\overset{s}{=} \int u^2\left(1-u^2\right)du = \int\left(u^2-u^4\right)du = \tfrac{1}{3}u^3 - \tfrac{1}{5}u^5 + C = \tfrac{1}{3}\sin^3 x - \tfrac{1}{5}\sin^5 x + C$$

9. $\int \sin^5(2t)\cos^2(2t)\,dt = \int \sin^4(2t)\cos^2(2t)\sin(2t)\,dt = \int\left[1-\cos^2(2t)\right]^2\cos^2(2t)\sin(2t)\,dt$

$$= \int -\tfrac{1}{2}\left(1-u^2\right)u^2\,du \qquad [u=\cos(2t), du = -2\sin(2t)dt]$$

$$= -\tfrac{1}{2}\int\left(u^4-2u^2+1\right)u^2\,du = -\tfrac{1}{2}\int\left(u^6-2u^4+u^2\right)du$$

$$= -\tfrac{1}{2}\left(\tfrac{1}{7}u^7 - \tfrac{2}{5}u^5 + \tfrac{1}{3}u^3\right) + C = -\tfrac{1}{14}\cos^7(2t) + \tfrac{1}{5}\cos^5(2t) - \tfrac{1}{6}\cos^3(2t) + C$$

13. $\int_0^\pi \cos^4(2t)\,dt = \int_0^\pi\left[\cos^2(2t)\right]^2 dt = \int_0^\pi\left[\tfrac{1}{2}(1+\cos(2\cdot 2t))\right]^2 dt$ [half-angle identity]

$$= \tfrac{1}{4}\int_0^\pi\left(1+2\cos 4t + \cos^2(4t)\right)dt = \tfrac{1}{4}\left[\tfrac{3}{2}t + \tfrac{1}{2}\sin 4t + \tfrac{1}{16}\sin 8t\right]_0^\pi = \tfrac{1}{4}\left[\left(\tfrac{3}{2}\pi+0+0\right)-0\right] = \tfrac{3}{8}\pi$$

17. $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta = \int\sqrt{\cos\theta}\,\sin^2\theta\sin\theta\,d\theta = \int(\cos\theta)^{1/2}(1-\cos^2\theta)\sin\theta\,d\theta$

$$\overset{c}{=} \int -u^{1/2}(1-u^2)\,du = \int\left(u^{5/2}-u^{1/2}\right)du$$

$$= \tfrac{2}{7}u^{7/2} - \tfrac{2}{3}u^{3/2} + C = \tfrac{2}{7}(\cos\theta)^{7/2} - \tfrac{2}{3}(\cos\theta)^{3/2} + C$$

21. $\int \sin^2 x \sin 2x\,dx = \int \sin^2 x(2\sin x\cos x)\,dx \overset{s}{=} \int 2u^3\,du = \tfrac{1}{2}u^4 + C = \tfrac{1}{2}\sin^4 x + C$

25. $\int \tan x \sec^3 x\,dx = \int \tan x \sec x \sec^2 x\,dx = \int u^2\,du \qquad [u = \sec x, du = \sec x \tan x\,dx]$

$$= \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\sec^3 x + C$$

29. Let $u = \tan x$. Then $du = \sec^2 x\,dx$, so

$$\int \tan^4 x \sec^6 x\,dx = \int \tan^4 x \sec^4 x\left(\sec^2 x\right)dx = \int \tan^4 x\left(1+\tan^2 x\right)^2\left(\sec^2 x\right)dx$$

$$= \int u^4\left(1+u^2\right)^2 du = \int\left(u^8+2u^6+u^4\right)du$$

$$= \tfrac{1}{9}u^9 + \tfrac{2}{7}u^7 + \tfrac{1}{5}u^5 + C = \tfrac{1}{9}\tan^9 x + \tfrac{2}{7}\tan^7 x + \tfrac{1}{5}\tan^5 x + C$$

33. $\int \tan^3 x \sec^6 x\,dx = \int \tan^3 x \sec^4 x \sec^2 x\,dx = \int \tan^3 x\left(1+\tan^2 x\right)\sec^2 x\,dx$

$$= \int u^3\left(1+u^2\right)^2 du \qquad \left[u = \tan x, du = \sec^2 x\,dx\right]$$

$$= \int u^3\left(u^4+2u^2+1\right)du = \int\left(u^7+2u^5+u^3\right)du$$

$$= \tfrac{1}{8}u^8 + \tfrac{1}{3}u^6 + \tfrac{1}{4}u^4 + C = \tfrac{1}{8}\tan^8 x + \tfrac{1}{3}\tan^6 x + \tfrac{1}{4}\tan^4 x + C$$

37. Let $u = x$, $dv = \sec x \tan x\,dx \Rightarrow du = dx$, $v = \sec x$. Then

$$\int x\sec x\tan x\,dx = x\sec x - \int \sec x\,dx = x\sec x - \ln\left|\sec x + \tan x\right| + C.$$

41. $\int_{\pi/4}^{\pi/2} \cot^5 t \csc^3 t\, dt = \int_{\pi/4}^{\pi/2} \cot^4 t \csc^2 t \csc t \cot t\, dt = \int_{\pi/4}^{\pi/2} \left(\csc^2 t - 1\right)^2 \csc^2 t \csc t \cot t\, dt$

$$= \int_{\sqrt{2}}^{1} -\left(u^2 - 1\right) u^2\, du \qquad [u = \csc t, du = -\csc t \cot t\, dt]$$

$$= \int_{1}^{\sqrt{2}} \left(u^6 - 2u^4 + u^2\right) du = \left[\tfrac{1}{7}u^7 - \tfrac{2}{5}u^5 + \tfrac{1}{3}u^3\right]_1^{\sqrt{2}} = \left(\tfrac{8}{7}\sqrt{2} - \tfrac{8}{5}\sqrt{2} + \tfrac{2}{3}\sqrt{2}\right) - \left(\tfrac{1}{7} - \tfrac{2}{5} + \tfrac{1}{3}\right)$$

$$= \frac{120 - 168 + 70}{105}\sqrt{2} - \frac{15 - 42 + 35}{105} = \frac{22}{105}\sqrt{2} - \frac{8}{105}$$

54. $\int_0^{\pi/2} \sin^2 \theta\, d\theta = \int_0^{\pi/2} \tfrac{1}{2}(1 - \cos 2\theta)\, d\theta = \tfrac{1}{2}\left[\theta - \tfrac{1}{2}\sin 2\theta\right]_0^{\pi/2} = \tfrac{1}{2}\left[\left(\tfrac{\pi}{2} - 0\right) - 0\right] = \tfrac{\pi}{4}$, option (**C**).

57. $\int_0^{\pi/4} \tan^2 \theta\, d\theta = \int_0^{\pi/4} \left(\sec^2 \theta - 1\right) d\theta = \left[\tan\theta - \theta\right]_0^{\pi/4} = \left(1 - \tfrac{\pi}{4}\right) - (0 - 0) = 1 - \tfrac{\pi}{4}$, option (**D**).

63. $f_{\text{ave}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 x \cos^3 x\, dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 x \left(1 - \cos^2 x\right) \cos x\, dx$

$$= \frac{1}{2\pi}\int_0^0 u^2\left(1 - u^2\right) du \quad [\text{where } u = \sin x] \quad = 0$$

64. (a) Let $u = \cos x$. Then $du = -\sin x\, dx \Rightarrow \int \sin x \cos x\, dx = \int u(-du) = -\tfrac{1}{2}u^2 + C = -\tfrac{1}{2}\cos^2 x + C_1$.

(b) Let $u = \sin x$. Then $du = \cos x\, dx \Rightarrow \int \sin x \cos x\, dx = \int u\, du = \tfrac{1}{2}u^2 + C = \tfrac{1}{2}\sin^2 x + C_2$.

(c) $\int \sin x \cos x\, dx = \tfrac{1}{2}\int \tfrac{1}{2}\sin 2x\, dx = -\tfrac{1}{4}\cos 2x + C_3$.

(d) Let $u = \sin x, dv = \cos x\, dx$. Then $du = \cos x\, dx, v = \sin x$, so $\int \sin x \cos x\, dx = \sin^2 x - \int \sin x \cos x\, dx$,

so, $\int \sin x \cos x\, dx = \tfrac{1}{2}\sin^2 x + C_4$.

Using $\cos^2 x = 1 - \sin^2 x$ and $\cos 2x = 1 - 2\sin^2 x$, we see that the answers differ only by a constant.