p. 597: 9-49 EOO, 51-55, 73-74

9. $\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx = \lim_{t\to\infty}\int_0^t (1+x)^{-1/4}\,dx = \lim_{t\to\infty}\left[\tfrac{4}{3}(1+x)^{3/4}\right]_0^t$ $\quad [u = 1+x, dv = dx]$

$= \lim_{t\to\infty}\left[\tfrac{4}{3}(1+t)^{3/4} - \tfrac{4}{3}\right] = \infty.$ Divergent

13. $\int_{-\infty}^0 2^r\,dr = \lim_{t\to-\infty}\int_t^0 2^r\,dr = \lim_{t\to-\infty}\left[\frac{2^r}{\ln 2}\right]_t^0 = \lim_{t\to-\infty}\left(\frac{1}{\ln 2} - \frac{2^t}{\ln 2}\right) = \frac{1}{\ln 2} - 0 = \frac{1}{\ln 2}.$ Convergent

17. $\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\left[e^{-1/x}\right]_1^t = \lim_{t\to\infty}\left(e^{-1/t} - e^{-1}\right) = 1 - e^{-1}.$ Convergent

21. $\int_2^\infty \frac{dv}{v^2+2v-3} = \lim_{t\to\infty}\int_2^t \frac{dv}{(v+3)(v-1)} = \lim_{t\to\infty}\int_2^t \left(\frac{-\frac{1}{4}}{v+3} - \frac{\frac{1}{4}}{v-1}\right)dv = \lim_{t\to\infty}\left[-\tfrac{1}{4}\ln|v+3| + \tfrac{1}{4}\ln|v-1|\right]_2^t$

$= \tfrac{1}{4}\lim_{t\to\infty}\left[\ln\left|\frac{v-1}{v+3}\right|\right]_2^t = \tfrac{1}{4}\lim_{t\to\infty}\left(\ln\frac{t-1}{t+3} - \ln\tfrac{1}{5}\right) = \tfrac{1}{4}(0 + \ln 5) = \tfrac{1}{4}\ln 5.$ Convergent

25. $\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^t$ $\quad$ [integration by parts with $u = \ln x, dv = (1/x^2)\,dx$]

$= \lim_{t\to\infty}\left(-\frac{\ln t}{t} - \frac{1}{t} + 1\right) \overset{H}{=} \lim_{t\to\infty}\left(-\frac{1/t}{1}\right) - \lim_{t\to\infty}\frac{1}{t} + \lim_{t\to\infty}1 = 0 - 0 + 1 = 1.$ Convergent

29. $\int_1^\infty \frac{dx}{\sqrt{x} + x\sqrt{x}} = \lim_{t\to\infty}\int_1^t \frac{dx}{\sqrt{x}(1+x)} = \lim_{t\to\infty}\int_1^{\sqrt{t}} \frac{1}{u(1+u^2)}(2u\,du)$ $\quad [u = \sqrt{x},\ du = 1/(2\sqrt{x})\,dx]$

$= \lim_{t\to\infty}\int_1^{\sqrt{t}} \frac{2}{(1+u^2)}\,du = \lim_{t\to\infty}\left[2\tan^{-1}u\right]_1^{\sqrt{t}} = \lim_{t\to\infty}2(\tan^{-1}\sqrt{t} - \tan^{-1}1)$

$= 2\left(\tfrac{\pi}{2} - \tfrac{\pi}{4}\right) = \tfrac{\pi}{2}.$ Convergent

33. $\int_{-1}^2 \frac{x}{(x+1)^2}\,dx = \lim_{t\to-1^+}\int_t^2 \frac{x}{(x+1)^2}\,dx = \lim_{t\to-1^+}\int_t^2 \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right]dx$ $\quad$ [partial fractions]

$= \lim_{t\to-1^+}\left[\ln|x+1| + \frac{1}{x+1}\right]_t^2 = \lim_{t\to-1^+}\left[\ln 3 + \tfrac{1}{3} - \left(\ln(t+1) + \frac{1}{t+1}\right)\right] = -\infty.$ Divergent

Note: To justify the last step, $\lim_{t\to-1^+}\left[\ln(t+1) + \frac{1}{x+1}\right] = \lim_{x\to0^+}\left(\ln x + \frac{1}{x}\right)$ $\begin{bmatrix}\text{substitute}\\ x \text{ for } t+1\end{bmatrix}$ $= \lim_{x\to0^+}\frac{x\ln x + 1}{x} = \infty$

since $\lim_{x\to0^+}(x\ln x) = \lim_{x\to0^+}\frac{\ln x}{1/x} \overset{H}{=} \lim_{x\to0^+}\frac{1/x}{-1/x^2} = \lim_{x\to0^+}(-x) = 0.$

37. There is an infinite discontinuity at $w = 2$.

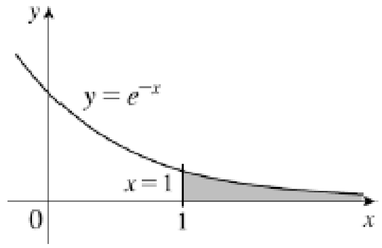
$\int_0^2 \frac{w}{w-2}\,dw = \lim_{t\to2^-}\int_0^t \left(1 + \frac{2}{w-2}\right)dw = \lim_{t\to2^-}\left[w + 2\ln|w-2|\right]_0^t = \lim_{t\to2^-}(t + 2\ln|t-2| - 2\ln 2) = -\infty$, so

$\int_0^2 \frac{w}{w-2}\,dw$ diverges, and hence, $\int_0^5 \frac{w}{w-2}\,dw$ diverges. Divergent

41. $\displaystyle\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{\sin\theta}}\,d\theta = \lim_{t\to 0^+}\int_t^{\pi/2} \frac{\cos\theta}{\sqrt{\sin\theta}}\,d\theta = \lim_{t\to 0^+}\left[\sqrt{\sin\theta}\right]_t^{\pi/2}$ $\quad [u=\sin\theta, du=\cos\theta\, d\theta]$

$\displaystyle = \lim_{t\to 0^+}\left(2-2\sqrt{\sin t}\right) = 2-0=2.$ Convergent

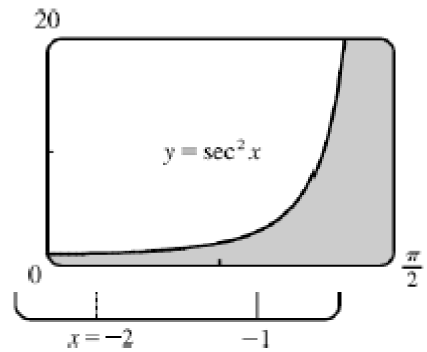
45. Area $\displaystyle = \int_1^\infty e^{-x}\,dx = \lim_{t\to\infty}\int_1^t e^{-x}\,dx = \lim_{t\to\infty}\left[-e^{-x}\right]_1^t = \lim_{t\to\infty}\left(-e^{-t}+e^{-1}\right) = 0+e^{-1} = 1/e.$



49. Area $\displaystyle = \int_0^{\pi/2}\sec^2 x\,dx = \lim_{t\to(\pi/2)^-}\int_0^t \sec^2 x\,dx = \lim_{t\to(\pi/2)^-}\left[\tan x\right]_0^t$

$\displaystyle = \lim_{t\to(\pi/2)^-}(\tan t - 0) = \infty.$

Infinite area



51. Only integral (**C**), $\displaystyle\int_1^2 \frac{dx}{x-1}$ is divergent. Integral (A),

$\displaystyle\int_0^\infty te^{-t}\,dt = 1$, Integral (B), $\displaystyle\int_0^\infty \frac{dx}{x^3+1} = \frac{2\pi\sqrt{3}}{9}$, and Integral (D), $\displaystyle\int_2^3 \frac{dx}{\sqrt{x-2}} = 2.$

52. $\displaystyle\int_1^\infty \frac{dx}{x^2+x} = \lim_{b\to\infty}\int_1^b\left(\frac{1}{x}-\frac{1}{x+1}\right)dx = \lim_{b\to\infty}\left(\ln x - \ln(x+1)\right)\Big|_1^b = \lim_{b\to\infty}\left(\ln b - \ln(b+1)\right) - \left(\ln 1 - \ln(2)\right)$

$\displaystyle = \lim_{b\to\infty}\left(\ln\left(\frac{b}{b+1}\right)+\ln 2\right) = \ln 1 + \ln 2 = \ln 2,$ which is choice (**B**).

53. Using the substitution $u = x^2$, $2x\,dx = du$,

$\displaystyle\int_1^\infty \frac{2x}{e^{x^2}} = \lim_{b\to\infty}\int_1^b \frac{2x}{e^{x^2}}\,dx = \lim_{b\to\infty}\int_1^b e^{-u}\,du = -\lim_{b\to\infty} e^{-u}\Big|_1^b = -\lim_{b\to\infty}\left(e^{-u}-e^{-1}\right) = e^{-1} = \frac{1}{e},$ option (**A**).

54. Using parts, $\displaystyle\int \frac{dx}{x(\ln x)^2} = -\frac{1}{\ln x}.$

Therefore, $\displaystyle\int_3^\infty \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\int_3^b \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\left(-\frac{1}{\ln x}\right)\Big|_3^b = \lim_{b\to\infty}\left(-\frac{1}{\ln b}+\frac{1}{\ln 3}\right) = \frac{1}{\ln 3},$ (**B**).

55. $\displaystyle\int_0^\infty \frac{dx}{e^{2x}} = \lim_{b\to\infty}\int_0^b e^{-2x}\,dx = -\frac{1}{2}\lim_{b\to\infty} e^{-2x}\Big|_0^b = -\frac{1}{2}\lim_{b\to\infty}\left(e^{-2b}-e^0\right) = \frac{1}{2},$ which is option (**B**).

73. (a) $f(x)=\dfrac{1}{x^2-x-6}=\dfrac{1}{(x+2)(x-3)}$ so $\lim\limits_{x\to 3^+} f(x)=\infty$, so $a=3$.

(b) Area $A=\lim\limits_{b\to\infty}\int_4^b f(x)\,dx=\lim\limits_{b\to\infty}\displaystyle\int_4^b\left(\frac{1}{5(x-3)}-\frac{1}{5(x+2)}\right)dx=\tfrac{1}{5}\cdot\lim\limits_{b\to\infty}\left(\ln(x-3)-\ln(x+2)\right)\Big|_4^b$

$$=\tfrac{1}{5}\cdot\lim_{b\to\infty}\left(\ln\left(\frac{b-3}{b+2}\right)-\ln\left(\frac{1}{6}\right)\right)=\tfrac{1}{5}\cdot\lim_{b\to\infty}\left(\ln(1)-\ln\left(\frac{1}{6}\right)\right)=\ln\left(6^{1/5}\right)=\ln\sqrt[5]{6}$$

(c) Area $B=\displaystyle\int_0^1\frac{1}{x^2-x-6}dx=\int_0^1\frac{1}{x^2-x-6}dx=\int_0^1\left(\frac{1}{5(x-3)}-\frac{1}{5(x+2)}\right)dx$

$$=\tfrac{1}{5}\cdot\left(\ln|x-3|-\ln(x+2)\right)\Big|_0^1=\tfrac{1}{5}\cdot\left(\ln 2-\ln 3-(\ln 3-\ln 2)\right)=\tfrac{2}{5}\left(\ln 2-\ln 3\right)$$

74. (a) Area $R=\int_0^\infty f(x)\,dx=\lim\limits_{b\to\infty}\int_0^\infty e^{-x}\,dx=\lim\limits_{b\to\infty}\left(-e^{-x}\right)\Big|_0^b=-\lim\limits_{b\to\infty}\left(e^{-b}-e^0\right)=e^0=1$

(b) Volume $S_1=\pi\int_0^\infty\left(\left(f(x)+3\right)^2-3^2\right)dx=\lim\limits_{b\to\infty}\pi\int_0^b\left(e^{-2x}+6e^{-x}+9-9\right)dx$

$$=\pi\lim_{b\to\infty}\left(-\tfrac{1}{2}e^{-2x}-6e^{-x}\right)\Big|_0^b=\pi\lim_{b\to\infty}\left[\left(-\tfrac{1}{2}e^{-2b}-6e^{-b}\right)-\left(-\tfrac{1}{2}e^0+6e^0\right)\right]=6.5\pi$$

(c) The equal sides of the isosceles triangle have length $\frac{1}{\sqrt{2}}e^{-x}$, so the area of each cross-section triangle is $\frac{1}{2}\left(\frac{1}{\sqrt{2}}e^{-x}\right)^2=\frac{1}{4}e^{-2x}$.

The volume of $S_2=\int_0^\infty\frac{1}{4}e^{-2x}\,dx=\dfrac{1}{4}\cdot\lim\limits_{b\to\infty}\left(-\dfrac{1}{2}\cdot e^{-2x}\right)\Big|_0^b=-\dfrac{1}{8}\cdot\lim\limits_{b\to\infty}\left(e^{-2b}-e^0\right)=\dfrac{1}{8}$.