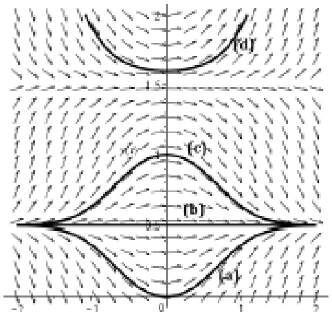
p. 615: 7-11, 13-16, 19-23 odd, 31-37, 40-43

7. 

It appears that the constant functions $y = 0.5$ and $y = 1.5$ are equilibrium solutions. Note that these two values of y satisfy the given differential equation $y' = x\cos \pi y$.

8. The function F depends on both x and y because the line segments along each horizontal are not parallel (do not have the same slope), nor do the line segments along each vertical line. The line segments in the fourth quadrant have negative slopes, so (C) is not true. However, the horizontal line segments occur for some x in [–1, 1], which indicates that $F = 0$ for some x in this interval. Thus, **D** is the true statement.

9. For $y > 0$, the given slope field indicates that F does not depend on y, and has horizontal tangents at $x = 1$. In addition, the slopes are positive for $x > 1$ and negative for $x < 1$, with the absolute value of the slopes decreasing as x approaches 1. The differential equation corresponding to the general solution $y = \pm(x-1)^2 + C$ would be $y = \pm 2(x-1)^2$. This means that option (**A**), $y = \pm(x-1)^2 + C$ could be the general solution for this slope field.
This slope field for $y > 0$ appears to be reflected across the y-axis to obtain the slope field for $y < 0$. Thus, for $y < 0$, the solution could be $y = -(x-1)^2 + C$.

10. The slope field indicates that the differential equation must involve both x and y, so possibilities (A) and (B) are eliminated. At the point (1, 4), the slope field has a line segment with a negative slope. This is true for option (**C**), but not for option (D).

11. In the given slope field, we see that the solution curve passing through (0, 1) has slopes that are positive and increasing, so (D) is true. For $y = 2e^x - x - 1 \Rightarrow \frac{dy}{dx} = 2e^2 - 1 = x + \left(2e^x - x - 1\right) = x + y$, so (C) is true. The solution passing through (0, 1) does appear to be asymptotic to the graph of $y = x - 1$, so (B) is true. However, if $y = x - 1$, then $x + y = 2x + 1 \neq 1 = \frac{dy}{dx}$, so statement **A** is not true.

13. $y' = 2 - y$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, **III** is the direction field for this equation. Note that for $y = 2$, $y' = 0$.

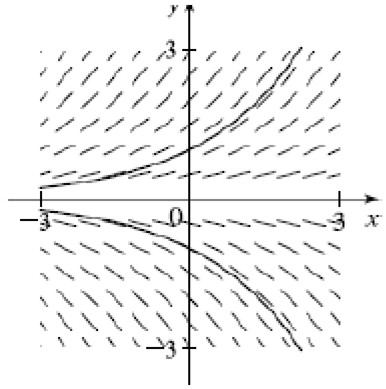
14. $y' = x(2 - y) = 0$ on the lines $x = 0$ and $y = 2$. Direction field **I** satisfies these conditions.

15. $y' = x + y - x = 0$ on the line $y = -x + 1$. Direction field **IV** satisfies these conditions. Observe also that on the line $y = -x$ we have $y' = -1$, which is true in IV.

16. $y' = \sin x \sin y = 0$ on the lines $x = 0$, $y = 0$ and $y' > 0\,0$ for $0 < x < \pi,\ 0 < y < \pi$. Direction field **II** satisfies these conditions.

19.

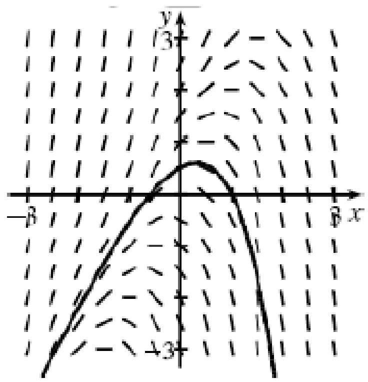
x	y	$y' = \frac{1}{2}y$
0	0	0
0	1	0.5
0	2	1
0	−3	−1.5
0	−2	−1



Note that the three solution curves sketched go through $(0,0), (0,1)$, and $(0,-1)$.

21.

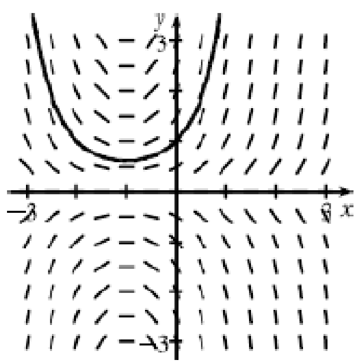
x	y	$y' = y - 2x$
−2	−2	2
−2	2	6
2	2	−2
2	−2	−6



Note that $y' = 0$ for any point on the line $y = 2x$. The slopes are positive to the left of the line and negative to the right of the line. The solution curve in the graph passes through (1, 0).

23.

x	y	$y' = y + xy$
0	±1	±2
1	±2	±4
−3	±2	±4



Note that $y' = y(x+1) = 0$ for any point on $y = 0$ or on $x = -1$. The slopes are positive when the factors y and have the same sign and negative when they have opposite signs. The solution curve in the graph passes through $(0,1)$.

31. $h = 0.5, x_0 = 1, y_0 = 0,$ and $F(x,y) = y - 2x.$

Note that $x_1 = x_0 + h = 1 + 0.5 = 1.5,\ x_2 = 2,$ and $x_3 = 2.5.$

$y_1 = y_0 + hF(x_0, y_0) = 0 + 0.5 \cdot F(1,0) = 0.5[0 - 2(1)] = -1$

$y_2 = y_1 + hF(x_1, y_1) = -1 + 0.5 \cdot F(1.5, -1) = -1 + 0.5\left[-1 - 2(1.5)\right] = -3$

$y_3 = y_2 + hF(x_2, y_2) = -3 + 0.5 \cdot F(2, -3) = -1 + 0.5\left[-3 - 2(2)\right] = -6.5$

$y_4 = y_3 + hF(x_3, y_3) = -6.5 + 0.5 \cdot F(2.5, -6.5) = -6.5 + 0.5\left[-6.5 - 2(2.5)\right] = -12.25$

32. $h = 0.2, x_0 = 0, y_0 = 1$, and $F(x, y) = x^2 y - \frac{1}{2}y^2$. $x_1 = x_0 + h = 0 + 0.2 = 0.2$, $x_2 = 0.4$, and $x_3 = 0.6$, $x_4 = 0.8$, and $x_5 = 1$.

$y_1 = y_0 + hF(x_0, y_0) = 1 + 0.2 \cdot F(0,1) = 1 + 0.2\left[0^2(1) - \frac{1}{2}(1)^2\right] = 1 + 0.2\left(-\frac{1}{2}\right) = 0.9$

$y_2 = y_1 + hF(x_1, y_1) = 0.9 + 0.2 \cdot F(0.2, 0.9) = 0.9 + 0.2\left[(0.2)^2(0.9) - \frac{1}{2}(0.9)^2\right] = 0.8262$

$y_3 = y_2 + hF(x_2, y_2) = 0.8262 + 0.2 \cdot F(0.4, 0.8262) = 0.784377756$

$y_4 = y_3 + hF(x_3, y_3) = 0.784377756 + 0.2F(0.6, 0.784377756) \approx 0.779328108$

$y_5 = y_4 + hF(x_4, y_4) = 0.779328108 + 0.2F(0.8, 0.779328108) \approx 0.818346876$

Thus, $y(1) \approx 0.818$

33. $F(x, y) = x + xy$, $h = 0.1$, $x_0 = 0, y_0 = 1$.

Note that $x_1 = x_0 + h = 0 + 0.1 = 0.1$, $x_2 = 0.2, x_3 = 0.3$ and $x_4 = 0.4$.

$y_1 = y_0 + h \cdot F(x_0, y_0) = 1 + 0.1 \cdot F(1, 0) = 1 + 0.1[1 + (0)(1)] = 1.1$.

$y_2 = y_1 + h \cdot F(x_1, y_1) = 1.1 + 0.1 \cdot F(0.1, 1.1) = 1.1 + 0.1[1.1 + (0.1)(1.1)] = 1.221$.

$y_3 = 1.221 + 0.1 \cdot F(0.2, 1.221) = 1.221 + 0.1[1.221 + (0.2)(1.221)] = 1.36752$.

$y_4 = 1.36752 + 0.1 \cdot F(0.3, 1.36752) = 1.36752 + 0.1[1.36752 + (0.3)(1.36752)] = 1.5452976$.

Thus, $y(0.4) \approx 1.5453$.

34. $F(x, y) = x + y$, $h = 0.5$, $x_0 = 0, y_0 = 1$. $y_1 = y_0 + hF(x_0, y_0) = 1 + 0.5 \cdot F(0,1) = 1 + 0.5(0 + 1) = 1.5$

$y_2 = y_1 + hF(x_1, y_1) = 1.5 + 0.5 \cdot F(0.5, 1.5) = 1.5 + 0.5(0.5 + 1.5) = 2.5$. Thus $y(1) \approx 2.5$, (**D**).

35. $F(x, y) = 4x(1 + y^2)$, $h = -0.25$, $x_0 = 1, y_0 = 0$.

$y_1 = y_0 + hF(x_0, y_0) = 0 - 0.25 \cdot F(1, 0) = 0 - 0.25 \cdot 4 \cdot 1 \cdot (1 + 0^2) = -1$

$y_2 = y_1 + hF(x_1, y_1) = -1 - 0.25 \cdot F(-0.75, -1) = -1 - 0.25 \cdot 4 \cdot (-0.75) \cdot (1 + (-1)^2) = -2.5$ Thus

$y\left(\frac{1}{2}\right) \approx 2.5$, choice (**D**).

36. $F(x, y) = \dfrac{0.36x}{y}$, $h = 0.2$, $x_0 = 0, y_0 = 6$. $y_1 = y_0 + hF(x_0, y_0) = 6 + 0.2 \cdot F(0,1) = 6 + 0.2\left(\dfrac{0.36 \cdot 0}{6}\right) = 6$

$y_2 = y_1 + hF(x_1, y_1) = 6 + 0.2 \cdot F(0.2, 6) = 6 + 0.2\left(\dfrac{0.36 \cdot 0.2}{6}\right) = 6.0024$

$y_3 = y_2 + hF(x_2, y_2) = 6.0024 + 0.2 \cdot F(0.4, 6.0024) = 6.0024 + 0.2\left(\dfrac{0.36 \cdot 0.4}{6.0024}\right) \approx 6.007198$.

Thus $f(0.6) \approx 6.007$, choice (**C**).

37. (a) $F(x,y)=\cos(x+y),\ h=0.2,\ x_0=0, y_0=0.$

Note that $x_1=x_0+h=0+0.2=0.2,\ x_2=0.4,$ and $x_3=0.6.$

$y_1=y_0+hF(x_0,y_0)=0+0.2\cdot F(0,0)=0.2\cos(0+0)=0.2$

$y_2=y_1+h\cdot F(x_1,y_1)=0.2+0.2\cdot F(0.2,0.2)=0.2+0.2\cos(0.4)\approx 0.3842121988.$

$y_3=y_2+h\cdot F(x_2,y_2)\approx 0.3842+0.2\cdot F(0.4,0.3842)\approx 0.52558011763.$ Thus, $y(0.6)\approx 0.5258.$

(b) Now use $h=0.1$. For $1\le n\le 6,\ x_n=0.n.$

$y_1=y_0+hF(x_0,y_0)=0+0.1\cos(0+0)=0.1$

$y_2=y_1+hF(x_1,y_1)=0.1+0.1\cos(0.2)\approx 0.1980$

$y_3\approx 0.1980+0.1\cos(0.3980)\approx 0.2920;\quad y_4\approx 0.2920+0.1\cos(0.5920)\approx 0.3733;$

$y_5\approx 0.3733+0.1\cos(0.7733)\approx 0.4448;\quad y_6\approx 0.4448+0.1\cos(0.9448)\approx 0.5034;$

Thus, $y(0.6)\approx 0.5034.$

40. (a) $y=6x+b\Rightarrow \frac{dy}{dx}=6.$ We need y to be a solution to the differential equation, so

$6=\frac{dy}{dx}=2y-12x=2(6x+b)-12x\Rightarrow 6=12x+2b-12x=2b\Rightarrow b=3.$

(b) $\frac{dy}{dx}=g'\Rightarrow g'(0,0)=2(0)-12(0)=0,$ thus, g has a critical point at $(0,0)$.

$\frac{d^2y}{dx^2}=2\frac{dy}{dx}-12=2(2y-12x)-12=4y-24x-12\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{(x,y)=(0,0)}=-12.$ Therefore g is concave down at this point and the graph of g must have a relative maximum at $(0,0)$.

41. $f(x)=y=-2x+4e^{-x}+c\Rightarrow f'(x)=\frac{dy}{dx}=-2-4e^{-x}\Rightarrow$

$\frac{dy}{dx}+y=-2-4e^{-x}+(-2x+4e^{-x}+c)=c-2-2x=-2x+(c-2).$ In order for f to be a solution of the differential equation we must have $c-2=0\Rightarrow c=2,$ which is option (**C**).

42. (a) $F(x,y)=x-y,\ h=0.2,\ x_0=0, y_0=1.$

$y_1=y_0+hF(x_0,y_0)=1+0.2\cdot F(0,1)=1+0.2(0-1)=0.8$

$y_2=y_1+hF(x_1,y_1)=0.8+0.2\cdot F(0.2,0.8)=0.8+0.2(0.2-0.8)=0.68$

$y_2=y_1+hF(x_1,y_1)=0.68+0.2\cdot F(0.4,0.68)=0.68+0.2(0.4-0.68)=0.624$

Euler's method gives an estimate of $f(0.6)\approx 0.624.$

(b) $f(x)=\frac{2}{e^x}+x+k\Rightarrow \frac{dy}{dx}=-2e^{-x}+1\Rightarrow \frac{dy}{dx}+y=-2e^{-x}+1+(2e^{-x}+x+k)=1+x+k\Rightarrow k=-1.$

43. (a) If f has a critical point at $x = \ln 2$, f must have a horizontal (or undefined) tangent line at this point. Therefore, $\left.\frac{dy}{dx}\right|_{(x,y)=(\ln 2,y)} = 2x - y\big|_{(x,y)=(\ln 2,y)} = 0 \Rightarrow 2(\ln 2) - y = 0 \Rightarrow y = 2\ln 2.$

(b) $\frac{dy}{dx} = 2x - y \Rightarrow \frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y.$ At the point $(\ln 2, 2\ln 2)$,
$2 - 2(\ln 2) + 2\ln y = 2 > 0,$ so this point is a relative minimum.

(c) $F(x,y) = 2x - y,\ h = -0.2,\ x_0 = 0, y_0 = 2.$

$y_1 = y_0 + hF(x_0, y_0) = 0 - 0.2 \cdot F(0,2) = 2 - 0.2(2 \cdot 0 + 2) = 2.4$

$y_2 = y_1 + hF(x_1, y_1) = 2.4 - 0.2 \cdot F(-0.2, 2.4) = 2.4 - 0.2(2 \cdot (-0.2) + 2.4) = 2.96$. At the point $(0,2)$, $\frac{d^2y}{dx^2} = 2 - (2 \cdot 0 - 2) = 4 > 0$ so the curve is concave up, $f(-0.4) \approx 2.96$ and will be an underestimate.