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5. $\frac{dy}{dx}=3x^2y^2 \Rightarrow \frac{dy}{y^2}=3x^2\,dx \Rightarrow \int\frac{dy}{y^2}=\int 3x^2\,dx\ \ [y\neq 0] \Rightarrow \int y^{-2}\,dy=\int 3x^2\,dx \Rightarrow -y^{-1}=x^3+C \Rightarrow$
$\frac{-1}{y}=x^3+C \Rightarrow y=\frac{-1}{x^3+C}$. $y=0$ is also a solution.

7. $xyy'=x^2+1 \Rightarrow xy\frac{dy}{dx}=x^2+1 \Rightarrow y\,dy=\frac{x^2+1}{x}dx\ \ [x\neq 0] \Rightarrow \int y\,dy=\int\left(x+\frac{1}{x}\right)dx \Rightarrow$
$\tfrac{1}{2}y^2=\tfrac{1}{2}x^2+\ln|x|+K \Rightarrow y^2=x^2+2\ln|x|+2k \Rightarrow y=\pm\sqrt{x^2+2\ln|x|+C}$, where $C=2K$.

9. $(e^y-1)y'=2+\cos x \Rightarrow (e^y-1)\frac{dy}{dx}=2+\cos x \Rightarrow (e^y-1)dy=(2+\cos x)\,dx \Rightarrow$
$\int(e^y-1)\,dy=\int(2+\cos x)\,dx \Rightarrow e^y-y=2x+\sin x+C$. We cannot solve explicitly for y.

14. $\frac{dz}{dt}+e^{t+z}=0 \Rightarrow \frac{dz}{dt}=-e^te^z \Rightarrow \int e^{-z}\,dz=\int e^t\,dt \Rightarrow -e^{-z}=-e^t+C \Rightarrow e^{-z}=e^t-C \Rightarrow$
$\frac{1}{e^z}=e^t-C \Rightarrow e^z=\frac{1}{e^t-C} \Rightarrow z=\ln\left(\frac{1}{e^t-C}\right) \Rightarrow z=-\ln\left(e^t-C\right)$

15. $\frac{dy}{dx}=ky \Rightarrow \int\frac{dy}{y}=\int k\,dx \Rightarrow \ln|y|=kx+C_1 \Rightarrow y=e^{kx+C_1}=Ce^{kx}$, equation (**C**).

16. $\int dy=\int 2x\,dx \Rightarrow y=f(x)=x^2+C$. $f(1)=3 \Rightarrow 3=(1)^2+C \Rightarrow C=1$. Therefore, $f(x)=x^2+2$, option (**B**).

17. $y=e^{x/2} \Rightarrow \frac{dy}{dx}=\tfrac{1}{2}e^{x/2} \Rightarrow 2\,dy=e^{x/2}\,dx \Rightarrow 2\frac{dy}{e^{x/2}}=dx \Rightarrow 2\frac{dy}{y}=dx$, which is choice, (**A**).

18. Observe that the graphs of equations (B), (C) and (D) go through (–1, 3), so they all satisfy the initial condition. $\frac{dy}{dx}=(y-1)(2x+1) \Rightarrow \int\frac{dy}{y-1}=\int(2x+1)dx \Rightarrow \ln|y-1|=x^2+x+K \Rightarrow$
$|y-1|=Ce^{x^2+x} \Rightarrow y=Ce^{x^2+x}+1$. The initial condition $y(-1)=3$ forces $C=2$, so equation (**D**) is the desired solution.

19. $f'(x)=x\sqrt{f(x)} \Rightarrow \frac{dy}{dx}=xy^{1/2} \Rightarrow \int y^{-1/2}\,dy=\int x\,dx \Rightarrow 2y^{1/2}=\tfrac{1}{2}x^2+K \Rightarrow \sqrt{y}=\tfrac{1}{4}x^2+C.$
$f(3)=25 \Rightarrow \sqrt{25}=\left(\tfrac{1}{4}\cdot 3^2+C\right) \Rightarrow \tfrac{11}{4}=C$. Therefore, $y=\left(\tfrac{1}{4}x^2+\tfrac{11}{4}\right)^2$, so $y(0)=\left(\tfrac{11}{4}\right)^2=\tfrac{121}{16}$, which is option (**D**).

20. $\frac{dy}{dx}=xe^y \Rightarrow \int e^{-y}dy=\int x\,dx \Rightarrow -e^{-y}=\tfrac{1}{2}x^2+C.\ y(0)=0 \Rightarrow -e^0=\tfrac{1}{2}(0)^2+C \Rightarrow$
$C=-1$, so $-e^{-y}=\tfrac{1}{2}x^2-1 \Rightarrow e^{-y}=-\tfrac{1}{2}x^2+1 \Rightarrow y=\ln\left(1-\tfrac{1}{2}x^2\right) \Rightarrow y=-\ln\left(1-\tfrac{1}{2}x^2\right)$.

21. $\frac{dy}{dx}=\frac{x\sin x}{y} \Rightarrow y\,dy=x\sin x\,dx \Rightarrow \int y\,dy=\int x\sin x\,dx \Rightarrow \tfrac{1}{2}y^2=-x\cos x+\sin x+C$ [by parts].
$y(0)=-1 \Rightarrow \tfrac{1}{2}(-1)^2=-0\cos 0+\sin 0+C \Rightarrow C=\tfrac{1}{2}$, so $\tfrac{1}{2}y^2=-x\cos x+\sin x+\tfrac{1}{2} \Rightarrow$
$y^2=-2x\cos x+2\sin x+1 \Rightarrow y=-\sqrt{-2x\cos x+2\sin x+1}$ because $y(0)=-1<0$.

22. $\frac{du}{dt}=\frac{2t+\sec^2 t}{2u}$, $u(0)=-5$. $\int 2u\,du=\int(2t+\sec^2 t)\,dt \Rightarrow u^2=t^2+\tan t+C$, where

$[u(0)]^2=0^2+\tan 0+C \Rightarrow C=(-5)^2=25$. Therefore, $u^2=t^2+\tan t+25$, so $u=\pm\sqrt{t^2+\tan t+25}$.

Because $u(0)=-5<0$, we must have, $u=-\sqrt{t^2+\tan t+25}$.

28. $\frac{dy}{dx}=\frac{x}{y} \Rightarrow y\,dy=x\,dx \Rightarrow \int y\,dy=\int x\,dx \Rightarrow \frac{1}{2}y^2=\frac{1}{2}x^2+C \Rightarrow y(0)=2 \Rightarrow \frac{1}{2}(2)^2=\frac{1}{2}(0)^2+C \Rightarrow$

$C=2$, so $\frac{1}{2}y^2=\frac{1}{2}x^2+2 \Rightarrow y^2=x^2+4 \Rightarrow y=\sqrt{x^2+4}$ since $y(0)=2>0$.

29. $f'(x)=xf(x)-x \Rightarrow \frac{dy}{dx}=xy-x \Rightarrow \frac{dy}{dx}=x(y-1) \Rightarrow \frac{dy}{y-1}=x\,dx \quad [y\neq 1] \Rightarrow$

$\int\frac{dy}{y-1}=\int x\,dx \Rightarrow \ln|y-1|=\frac{1}{2}x^2+C$. $f(0)=2 \Rightarrow \ln|2-1|=\frac{1}{2}(0)^2+C \Rightarrow C=0$, so

$\ln|y-1|=\frac{1}{2}x^2 \Rightarrow |y-1|=e^{x^2/2} \Rightarrow y-1=e^{x^2/2}$ [since $f(0)=2$] $\Rightarrow y=e^{x^2/2}+1$

34.

$\frac{dy}{dx}=x\sin x \Rightarrow \int dy=\int x\sin x\,dx \Rightarrow y=-x\cos x+\sin x+C$ [by parts].

$f(1)=-1\cos 1+\sin 1+C=3 \Rightarrow C=2.699$. Therefore, $f(2)=4.440$ (**?**)

42. (a) For this differential equation, when $x=0 \Rightarrow \frac{dy}{dx}-0=0 \Rightarrow \frac{dy}{dx}=0$, so the slope of the tangent line is zero. Therefore, when $x=0$, the tangent line must be horizontal.

(b) $\frac{dy}{dx}-2xy=x \Rightarrow \frac{dy}{dx}=x(2y+1) \Rightarrow \int\frac{dy}{2y+1}=\int x\,dx \Rightarrow \frac{1}{2}\ln|2y+1|=\frac{1}{2}x^2+C \Rightarrow$

$\ln|2y+1|=x^2+C \Rightarrow 2y+1=e^{x^2+C} \Rightarrow y+1=\frac{1}{2}Ae^{x^2} \Rightarrow y=\frac{1}{2}\left(Ae^{x^2}-1\right)$

(c) $y(0)=0 \Rightarrow 0=\frac{1}{2}\left(Ae^0-1\right) \Rightarrow A=1$, so the solution is $y=\frac{1}{2}(e^{x^2}-1)$

43. $f'(x)=2x[f(x)]^2 \Rightarrow \frac{dy}{dx}=2xy^2 \Rightarrow \int y^{-2}dy=\int 2x\,dx \Rightarrow -y^{-1}=x^2+C \Rightarrow y=-\left(\frac{1}{x^2+C}\right)$.

$f(0)=2=-\left(\frac{1}{0^2+C}\right) \Rightarrow C=-\frac{1}{2} \Rightarrow y=\frac{-2}{2x^2-1}$. Therefore, $f(2)=\frac{-2}{2(2)^2-1}=-\frac{2}{7}$, **(B)**.

44. (a) At this point, the tangent line has slope $\left.\frac{dy}{dx}\right|_{(x,y)=(0,1)} = 4-1=-3$. Therefore, the equation of the tangent line at (0, 1) is $y-1=3(x-0)$, or $y=3x+1$. Using this tangent line we can approximate $f(1.5)\approx 3(1.5)+1=5.5$.

(b) $\frac{dy}{dx}=4-y \Rightarrow \int\frac{dy}{4-y}=\int dx \Rightarrow \ln|4-y|=x+C \Rightarrow |4-y|=Ae^{x} \Rightarrow 4-y=Ae^{x} \Rightarrow$

$f(x)=4-Ae^{x}$. Then $\lim_{x\to 0}\frac{f(x)}{6x}=\lim_{x\to 0}\frac{4-Ae^{x}}{6x}=\frac{4-A}{\lim_{x\to 0}6}=\infty$, so the limit does not exist.

(c) From (b), the general solution is $y=4\pm Ae^{x}$. $f(0)=1 \Rightarrow 1=4-A\cdot e^{0} \Rightarrow A=3$. Thus, $y=f(x)=4-3e^{-x}$.

48. (a) When $t = 1$, the tangent line has slope $\frac{dW}{dt}=\frac{1}{5}(80-30)=\frac{50}{5}=10 \Rightarrow$ the equation of the tangent line at (0, 30) is $y=10t+30$. We can use the tangent line to approximate $W(1)\approx 10+30=40$ g.

(b) $\frac{d^2W}{dt^2}=-\frac{1}{5}\cdot\frac{dW}{dt}=-\frac{1}{5}\left[\frac{1}{5}(80-W)\right]=-\frac{1}{25}(80-W)$; Because the second derivative is negative at the point (0, 30), the graph of W is concave down, so the tangent line lies above the curve and the tangent line estimate is an overestimate.

(c) $\frac{dW}{dt}=\frac{1}{5}(80-W) \Rightarrow \int\frac{dW}{(80-W)}=\int\frac{1}{5}dt \Rightarrow \ln|80-W|=\frac{1}{5}t+C \Rightarrow |80-W|=Ae^{0.2t} \Rightarrow$

$80-W=Ae^{0.2t} \Rightarrow W(t)=80-Ae^{0.2t}$.

$W(0)=30 \Rightarrow 30=80+Ae^{0} \Rightarrow A=-50 \Rightarrow W(t)=80-50e^{0.2t}$.

49. (a) $\frac{dy}{dx}=\frac{2}{xy}=2(xy)^{-1} \Rightarrow \frac{d^2y}{dx^2}=-2(xy)^{-2}\left(x\frac{dy}{dx}+y\right)=\frac{-2}{x^2y^2}\left(x\frac{2}{xy}+y\right)=\frac{-2}{x^2y^2}\left(\frac{2+y^2}{y}\right)=\frac{-4-2y^2}{x^2y^3}$.

(b) At the point (1, 3), the tangent line has slope $\frac{2}{1\cdot 3}=\frac{2}{3}$. The equation of the tangent line at this point is $y=\frac{2}{3}(x-1)+3=\frac{2}{3}x+\frac{7}{3}$. We can approximate $f(1.2)\approx\frac{2}{3}(1.2)+\frac{7}{3}=\frac{47}{15}\approx 3.133$

(c) If $f(x)>0$ for $1\le x\le 1.5$, by (a), $f''(x)=\frac{-4-2y^2}{x^2y^3}<0$ on this interval, which means that the curve is concave down on this interval, and the tangent line lies above the curve, so the tangent line estimate is an overestimate.

(d) $\frac{dy}{dx}=\frac{2}{xy} \Rightarrow \int y\,dy=\int 2x^{-1}\,dx \Rightarrow \frac{1}{2}y^2=2\ln|x|+C \Rightarrow y^2=4\ln|x|+c$.

$y(1)=3 \Rightarrow 3^2=4\ln|1|+c \Rightarrow 9=c \Rightarrow y^2=4\ln x+9 \Rightarrow y=\sqrt{4\ln x+9}$.

50. $x^2 f'(x) = f(x) \Rightarrow x^2 \dfrac{dy}{dx} = y \Rightarrow \displaystyle\int \frac{dy}{y} = \int \frac{dx}{x^2} \Rightarrow \ln|y| = -\frac{1}{x} + C \Rightarrow y = ce^{-1/x}$, and $f(1) = \dfrac{2}{e} \Rightarrow$

$\dfrac{2}{e} = \dfrac{c}{e} \Rightarrow c = 2,\ y = 2e^{-1/x}$. For this function, $f(-1) = 2e,\ f(2) = \dfrac{2}{\sqrt{e}},\ \lim\limits_{x \to 0} f(x)$ does not exist because the limit from the left does not equal the limit from the right. However, $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{2}{e^{1/x}} = 2$, so only (**B**) is true.

51. (a) $\dfrac{dy}{dt} = k(y - 70) \Rightarrow \displaystyle\int \frac{dy}{y - 70} = \int k\,dt \Rightarrow \ln|y - 70| = kt + C_1 \Rightarrow y - 70 = Ce^{kt} \Rightarrow y = Ce^{kt} + 70.$

$y(0) = 88 \Rightarrow 88 = C + 70 \Rightarrow C = 18 \Rightarrow y = 18e^{kt}.$

$y(2) = 85 \Rightarrow 85 = 18e^{2k} + 70 \Rightarrow 15/18 = e^{2k} \Rightarrow \ln(15/18) = 2k \Rightarrow k = \frac{1}{2}\ln\frac{5}{6}$

(b) $y = 70 + 18e^{(1/2)\ln(5/6)t}$

(c) $75 = 70 + 18e^{(1/2)\ln(5/6)t} \Rightarrow \frac{5}{18} = e^{0.5\ln(5/6)t} \Rightarrow 2 \cdot \ln\left(\frac{5}{18}\right) = \ln\left(\frac{5}{6}\right)t \Rightarrow t = \dfrac{2 \cdot \ln\left(\frac{5}{18}\right)}{\ln\left(\frac{5}{6}\right)} \approx 14.051$ min.