Logistic Worksheet

① $\frac{dP}{dt} = \frac{3P}{2}\left(1 - \frac{P}{60}\right) \rightarrow L = 60$

a) $P = 60$

b) $P = 30$

c) $\frac{dP}{dt} = \frac{3(30)}{2}\left(1 - \frac{30}{60}\right) = \frac{45}{2}$

② $P(0) = 40$

$P(5) = 104$

$P = \frac{4000}{1 + be^{-kt}}$

$P(0) = 40 \rightarrow 40 = \frac{4000}{1 + be^{0}}$

$40 = \frac{4000}{1 + b}$

$1 + b = 100$

$b = 99$

$P(5) = 104 \rightarrow 104 = \frac{4000}{1 + 99e^{-k \cdot 5}}$

$1 + 99e^{-5k} = \frac{4000}{104}$

$99e^{-5k} = \frac{4000}{104} - 1$

$e^{-5k} = \frac{\frac{4000}{104} - 1}{99}$

$-5k = \ln\left(\frac{\frac{4000}{104} - 1}{99}\right)$

$k = .194$

$\boxed{P = \frac{4000}{1 + 99e^{-.194t}}}$

$\boxed{P(15) = 628.538}$

③ a) $P = \frac{1000}{1 + e^{4.8}e^{-.7t}} \rightarrow L = 1000$

$k = .7$

b) $P(0) = \frac{1000}{1 + e^{4.8}} = 8.163$ means 8 rabbits were initially released.

④ a) $P = \frac{200}{1 + e^{5.3}e^{-t}} \rightarrow L = 200$

$k = 1$

b) $P(0) = \frac{200}{1 + e^{5.3}} = .993$ means that only 1 student was infected on Day

⑤ $P(0) = 6$

$\frac{dP}{dt} = .0015(150)P\left(1 - \frac{P}{150}\right)$

$= .225P\left(1 - \frac{P}{150}\right) \rightarrow L = 150$

$k = .225$

a) $P = \frac{150}{1 + be^{-.225t}} \rightarrow P(0) = \frac{150}{1 + b} = 6$

$1 + b = \frac{150}{6}$

$b = 24$

$\boxed{P = \frac{150}{1 + 24e^{-.225t}}}$

b) $100 = \frac{150}{1 + 24e^{-.225t}}$

$t = 17.205$

$125 = \frac{150}{1 + 23e^{-.225t}}$

$t = 21.459$

⑥ $P(0) = 28$ $\quad \frac{dP}{dt} = .0004(250)P\left(1 - \frac{P}{250}\right)$

$= .1P\left(1 - \frac{P}{250}\right) \rightarrow \begin{matrix} L = 250 \\ k = .1 \end{matrix}$

a) $P = \frac{250}{1+be^{-.1t}}$ $\quad P(0) = \frac{250}{1+b} = 28$ $\quad \boxed{P = \frac{250}{1+7.929e^{-.1t}}}$

$1+b = \frac{250}{28}$

$b = 7.929$

b) Fastest when $P = 125 \rightarrow 125 = \frac{250}{1+7.929e^{-.1t}}$

$t = 20.705$

⑦ $\frac{dP}{dt} = .05P\left(1 - \frac{P}{100}\right) \rightarrow L = 100$

⑧ B

⑨ $\frac{dP}{dt} = kP\left(1 - \frac{P}{200}\right) = kP - \frac{k}{200}P^2$

$\rightarrow$ coefficient on $P$ is $-200$ times bigger than coefficient on $P^2$, so the answer is A.

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7. (a) Comparing the given equation, $\frac{dP}{dt} = 0.04P\left(1 - \frac{P}{1200}\right)$, to $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$, we see that the carrying capacity is $M = 1200$ and the value of $k$ is 0.04.

(b) By Equation (12) the solution of the equation is $P(t) = \frac{M}{1+Ae^{-kt}}$, where $A = \frac{M - P_0}{P_0}$. Because $P(0) = P_0 = 60$, we have $A = \frac{1200-60}{60} = 19$, and hence, $P(t) = \frac{1200}{1+19e^{-0.04t}}$.

(c) The population after 10 weeks is $P(10) = \frac{1200}{1+19e^{-0.4}} \approx 87$.

8. (a) $\frac{dP}{dt} = 0.02P - 0.0004P^2 = 0.02P(1 - 0.02P) = 0.02P\left(1 - \frac{P}{50}\right)$, so carrying capacity is $M = 50$ and $k = 0.02$

(b) Solution would be $P(t) = \frac{M}{1+Ae^{-kt}} = \frac{50}{1+Ae^{-0.02t}}$. Since $P(0) = 40$, we have $40 = \frac{50}{1+Ae^{-0.02(0)}} \Rightarrow$ $40 = \frac{50}{1+A} \Rightarrow A = 0.25$. So $P(t) = \frac{50}{1+0.25e^{-0.02t}}$.

(c) $P(10) = \frac{50}{1+0.25e^{-0.02(10)}} = 41.505$

9. $\frac{dy}{dt} = 0.01y\left(1 - \frac{y}{1000}\right)$ is a logistic differential equation with carrying capacity $M = 1000$, so it has an inflection point at $M/2 = 500$, option (**D**).

10. The general solution of $\frac{dy}{dx} = 0.2y$ is $y = f(x) = Ae^{0.2x}$ and $f(0) = 5 \Rightarrow 5 = Ae^{0} \Rightarrow \quad A = 5 \Rightarrow$ $f(x) = 5e^{0.2x}$, choice (**A**).

17. Let $P(t)$ represent the amount of the substance at time $t$. This is an exponential decay initial value problem with $\frac{dP}{dt} = kt \Rightarrow P(t) = 3e^{-kt}$. The half-life is 250 years, so $P(250) = 3e^{-250k} = 1.5 \Rightarrow e^{250k} = \frac{1}{2} \Rightarrow 250k = \ln\frac{1}{2} \Rightarrow k = -\frac{\ln 2}{250} = -0.004\ln 2$. Therefore, a formula for the amount of substance, in grams, left after $t$ year is (**C**), $P = 3e^{(-0.004\ln 2)t}$.