Derivative Review

p. 286: 1-39 odd, 49, 56

1. $y=(x^2+x^3)^4 \Rightarrow y'=4(x^2+x^3)^3(2x+3x^2)=4(x^2)^3(1+x)^3x(2x+3)=4x^7(x+1)^3(3x+2)$

3. $y=\dfrac{x^2-x+2}{\sqrt{x}}=x^{3/2}-x^{1/2}+2x^{-1/2} \Rightarrow y'=\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}-x^{-3/2}=\dfrac{3}{2}\sqrt{x}-\dfrac{1}{2\sqrt{x}}-\dfrac{1}{\sqrt{x^3}}$

5. $y=x^2\sin\pi x \Rightarrow y'=x^2(\cos\pi x)\pi+(\sin\pi x)(2x)=x(\pi x\cos\pi x+2\sin\pi x)$

7. $y=\dfrac{t^4-1}{t^4+1} \Rightarrow y'=\dfrac{(t^4+1)4t^3-(t^4-1)4t^3}{(t^4+1)^2}=\dfrac{4t^3[(t^4+1)-(t^4-1)]}{(t^4+1)^2}=\dfrac{8t^3}{(t^4+1)^2}$

9. $y=\ln(x\ln x) \Rightarrow y'=\dfrac{1}{x\ln x}(x\ln x)'=\dfrac{1}{x\ln x}\left(x\cdot\dfrac{1}{x}+\ln x\cdot 1\right)=\dfrac{1+\ln x}{x\ln x}$

Another method: $y=\ln(x\ln x)=\ln x+\ln(\ln x) \Rightarrow y'=\dfrac{1}{x}+\dfrac{1}{\ln x}\cdot\dfrac{1}{x}=\dfrac{\ln x+1}{x\ln x}$

11. $y=\sqrt{x}\cos\sqrt{x} \Rightarrow y'=\sqrt{x}\left[-\sin\sqrt{x}(\frac{1}{2}x^{-1/2})\right]+\cos\sqrt{x}(\frac{1}{2}x^{-1/2})=\dfrac{\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x}}{2\sqrt{x}}$

13. $y=\dfrac{e^{1/x}}{x^2} \Rightarrow y'=\dfrac{x^2\left(e^{1/x}\right)'-e^{1/x}(x^2)'}{(x^2)^2}=\dfrac{x^2e^{1/x}(-1/x^2)-e^{1/x}(2x)}{x^4}=\dfrac{-e^{1/x}(1+2x)}{x^4}$

15. $\dfrac{d}{dx}(y+x\cos y)=\dfrac{d}{dy}\left(x^2y\right) \Rightarrow y'+x(-\sin y\cdot y')+\cos y=x^2y'+y\cdot 2x \Rightarrow$

$y'-x\sin y\cdot y'-x^2y'=2xy-\cos y \Rightarrow (1-x\sin y-x^2)y'=2xy-\cos y \Rightarrow y'=\dfrac{2xy-\cos y}{1-x\sin y-x^2}$

17. $y=\sqrt{\arctan x} \Rightarrow y'=\dfrac{1}{2}(\arctan x)^{-1/2}\dfrac{1}{1+x^2}=\dfrac{1}{2\sqrt{\arctan}(1+x^2)}$

19. $y=\tan\left(\dfrac{t}{1+t^2}\right) \Rightarrow y'=\sec^2\left(\dfrac{t}{1+t^2}\right)\cdot\dfrac{(1+t^2)(1)-t(2t)}{(1+t^2)^2}=\dfrac{1-t^2}{(1+t^2)^2}\sec^2\left(\dfrac{t}{1+t^2}\right)$

21. $y=3^{x\ln x} \Rightarrow y'=3^{x\ln x}(\ln 3)\cdot\left(x\cdot\dfrac{1}{x}+\ln x\cdot 1\right)=3^{x\ln x}(\ln 3)(1+\ln x)$

23. $y=(1-x^{-1})^{-1} \Rightarrow y'=-1(1-x^{-1})^{-2}[-(-x^{-2})]=-(1-1/x)^{-2}x^{-2}=-((x-1)/x)^{-2}x^{-2}=-(x-1)^{-2}$

25. $\sin(xy)=x^2-y \Rightarrow \cos(xy)(xy'+y)=2x-y' \Rightarrow x\cos(xy)y'+y'=2x-y\cos(xy) \Rightarrow$

$y'[x\cos(xy)+1]=2x-y\cos(xy) \Rightarrow y'=\dfrac{2x-y\cos(xy)}{x\cos(xy)+1}$

27. $y=\log_5(1+2x) \Rightarrow y'=\dfrac{2}{(1+2x)\ln 5}$

29. $y=\ln\sin x-\frac{1}{2}\sin^2 x \Rightarrow y'=\dfrac{1}{\sin x}\cdot\cos x-\frac{1}{2}\cdot 2\sin x\cdot\cos x=\cot x-\sin x\cos x$

31. $y=x\tan^{-1}(4x) \Rightarrow y'=x\cdot\dfrac{1}{1+(4x)^2}\cdot 4+\tan^{-1}(4x)\cdot 1=\dfrac{4x}{1+16x^2}+\tan^{-1}(4x)$

33. $y=\ln|\sec 5x+\tan 5x| \Rightarrow$

$$y'=\frac{1}{\sec 5x+\tan 5x}\left(\sec 5x\tan 5x\cdot{}+\sec^2 5x\cdot 5\right)=\frac{5\sec 5x(\tan 5x+\sec 5x)}{\sec 5x+\tan 5x}=5\sec 5x$$

35. $y=\cot(3x^2+5) \Rightarrow y'=-\csc^2(3x^2+5)(6x)=-6x\csc^2(3x^2+5)$

37. $y=\sin\left(\tan\sqrt{1+x^3}\right) \Rightarrow y'=\cos\left(\tan\sqrt{1+x^3}\right)\left(\sec^2\sqrt{1+x^3}\right)[3x^2/(2\sqrt{1+x^3})]$

39. $y=\tan^2(\sin\theta)=[\tan(\sin\theta)]^2 \Rightarrow y'=2[\tan(\sin\theta)]\cdot\sec^2(\sin\theta)\cdot\cos\theta$

49. $x^6+y^6=1 \Rightarrow 6x^5+6y^5y'=0 \Rightarrow y'=-x^5/y^5 \Rightarrow$

$$y''=-\frac{y^5(5x^4)-x^5(5y^4y')}{y^{10}}=-\frac{5x^4y^4[y-x(-x^5/y^5)]}{y^{10}}=-\frac{5x^4[(y^6+x^6)/y^5)]}{y^{10}}=-\frac{5x^4}{y^{11}}$$

56. $x^2+4xy+y^2=13 \Rightarrow 2x+4(xy'+y)+2yy'=0 \Rightarrow x+2xy'+2y+yy'=0 \Rightarrow$

$$2xy'+yy'=-x-2y \Rightarrow y'(2x+y)=-x-2y \Rightarrow y'\frac{-x-2y}{2x+y}.$$

At $(2,1)$, $y'=\frac{-2-2}{4+1}=-\frac{4}{5}$, so an equation of the tangent line is $y=-\frac{4}{5}(x-2)+1$, or $y=-\frac{4}{5}x+\frac{13}{5}$.

The slope of the normal line is $\frac{5}{4}$, so an equation of the normal line is $y=\frac{5}{4}(x-2)+1$, or $y=\frac{5}{4}x-\frac{3}{2}$.