# MVT, IVT, EVT

**For problems 1-8, determine if the Mean Value Theorem applies to the function on the given interval. If it does, find the $c$-value. If it doesn't, explain why not.**

1. $f(x)=|x|$ $\quad [-1,3]$ $\quad$ not diff. at $x=0$, so does not apply

2. $f(x)=x^2-2x$ $\quad [1,3]$ $\quad$ diff., so MVT applies

$\frac{f(3)-f(1)}{3-1} = \frac{3-(-1)}{2} = 2$ $\qquad f'(x)=2x-2=2$

$2x=4$

$x=2$

3. $f(x)=x^2-3x+2$ $\quad [1,2]$ $\quad$ diff., so MVT applies

$\frac{f(2)-f(1)}{2-1} = \frac{0-0}{1} = 0$ $\qquad f'(x)=2x-3=0$

$x=\frac{3}{2}$

4. $f(x)=x^{2/3}$ $\quad [-2,2]$ $\quad f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3x^{1/3}} \rightarrow$ undef. at $x=0$

not diff. at $x=0$, so MVT does not apply

5. $f(x)=\frac{1}{x-4}$ $\quad [2,6]$ $\quad$ undef. at $x=4$, so not cont.

$\Rightarrow$ MVT does not apply

6. $f(x)=\frac{x^2-x}{x}$ $\quad [-1,1]$ $\quad$ undef. at $x=0$, so not cont.

$\Rightarrow$ MVT does not apply

7. $f(x)=\sin x$ $\quad [0,\pi]$ $\quad f'(x)=\cos x=0$

$\frac{f(\pi)-f(0)}{\pi-0} = \frac{0-0}{\pi} = 0$ $\qquad x=\frac{\pi}{2}$

8. $f(x)=\tan x = \frac{\sin x}{\cos x}$ $\quad [0,\pi]$ $\quad$ undef. at $x=\frac{\pi}{2}$, so not cont.

$\Rightarrow$ MVT does not apply

For problems 9-13, determine if the Intermediate Value Theorem would guarantee a $c$-value on the given interval.

9. $f(x)=x^2+x-1$ $\quad f(c)=11$ $\quad [0,5]$

$f(0)=-1$

$f(5)=29$

$f(0)<11<f(5)$ and $f$ is cont.

$\Rightarrow$ IVT applies

10. $f(x)=\dfrac{x}{x-1}$ $\quad f(c)=1$ $\quad [0,2]$

Not continuous at $x=1$, so IVT does not apply

11. $f(x)=|x|$ $\quad f(c)=3$ $\quad [-4,1]$

$f(-4)=4$

$f(1)=1$

$f(1)<3<f(-4)$ and $f$ is cont

$\Rightarrow$ IVT applies

12. $f(x)=\begin{cases} x & x\le 1 \\ 3 & x>1 \end{cases}$ $\quad f(c)=2$ $\quad [0,4]$

$\left.\begin{array}{l} \lim_{x\to 1^+} f(x)=3 \\ \lim_{x\to 1^-} f(x)=1 \end{array}\right\}$ $f$ is not cont. at $x=1$, so IVT does not apply

13. $f(x)=\dfrac{x^2+x}{x-1}$ $\quad f(c)=6$ $\quad \left[\frac{5}{2},4\right]$

$f\left(\frac{5}{2}\right)=\frac{35}{6}=5.833$

$f(4)=\frac{20}{3}=6.667$

$f\left(\frac{5}{2}\right)<6<f(4)$ and $f$ is cont. ($x=1$ not on interval), so IVT applies

For problems 14-16, find the $c$-values for the given problem.

14. Problem 9

$x^2+x-1=11$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$\cancel{x=-4}$ $\quad \boxed{x=3}$

15. Problem 11

$|x|=3$

$\cancel{x=3}$ $\quad \boxed{x=-3}$

16. Problem 13

$\dfrac{x^2+x}{x-1}=6$

$x^2+x=6(x-1)$

$x^2+x=6x-6$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$\cancel{x=2}$ $\quad \boxed{x=3}$

For Problems 17-21, use the table below with selected values of the twice differentiable function $k$. Reach each explanation and decide whether you would apply IVT, EVT, or MVT.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $k(x)$ | 5 | 2 | -4 | -1 | 3 | 2 | 0 |

17. Since $k$ is differentiable, it is also continuous. Since $k(6)=2$ and $k(7)=0$, and since 1 is between 2 and 0, it follows by IVT that $k(c)=1$ for some $c$ between 6 and 7.

18. Since $k$ is differentiable and, therefore, also continuous, and since $\frac{k(3)-k(2)}{3-2}=-6$, it follows by MVT that $k'(c)=-6$ for some $c$ in the interval $(2,3)$.

19. There must be a minimum value for $k$ at some $r$ in $[1,7]$, because $k$ is differentiable and, therefore, also continuous. Hence the EVT applies.

20. There must be some value $a$ in $(2,6)$ for which $k'(a)=0$, because $k(2)=k(6)$, and since $k$ is differentiable, the MVT applies.

21. Since $k$ is differentiable, the MVT guarantees some $a$ in $(4,5)$ for which $k'(a)=4$ and also some $b$ in $(5,6)$ for which $k'(b)=-1$. Then since $k'$ is differentiable, and therefore also continuous, it follows by the IVT applied to $k'$ that $k'(c)=0$ for some $c$ in $(a,b)$ and therefore in $(4,6)$.