<u>Other Applications</u>

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7. $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{1}{4}\left(\frac{2}{3}\cdot 8\right) = \frac{4}{3}$

9. $g_{\text{ave}} = \frac{1}{b-a}\int_a^b g(t)\,dt = \frac{1}{3-1}\int^{2} \frac{t}{\sqrt{3+t^2}}\,dt = \frac{1}{2}\left[(3+t^2)^{1/2}\right]_1^3 = \frac{1}{2}\left(2\sqrt{3}-2\right) = \sqrt{3}-1$

11. $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{1-(-1)}\int_{-1}^{1} \frac{x^2}{(x^3+3)^2}\,dx = \frac{1}{2}\int_2^4 \frac{1}{u^2}\cdot\left(\frac{1}{3}du\right)$ $\left[\begin{array}{c} u = x^3+3 \\ du = 3x^2\,dx \end{array}\right] = \frac{1}{6}\left[-\frac{1}{u}\right]_2^4 = \frac{1}{6}\left(-\frac{1}{4}+\frac{1}{2}\right) = \frac{1}{24}$

13. $h_{\text{ave}} = \frac{1}{b-a}\int_a^b h(u)\,du = \frac{1}{5-1}\int^{5} \frac{\ln u}{u}\,du = \frac{1}{4}\int_0^{\ln 5} y\,dy$ $\left[\begin{array}{c} y = \ln u \\ dy = 1/u\,du \end{array}\right] = \frac{1}{4}\left[\frac{1}{2}y^2\right]_0^{\ln 5} = \frac{1}{8}(\ln 5)^2$

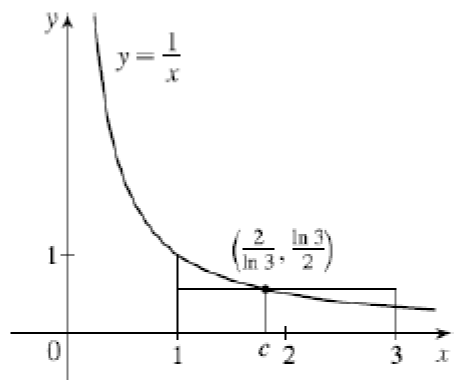
15. (a) $f_{\text{ave}} = \frac{1}{3-1}\int^{2} \frac{1}{x}\,dx = \frac{1}{2}\left[\ln|x|\right]_1^3$

$= \frac{1}{2}\left[\ln 3 - \ln 1\right] = \frac{1}{2}\ln 3$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow \frac{1}{c} = \frac{1}{2}\ln 3 \Leftrightarrow$

$c = \frac{2}{\ln 3} \approx 1.820.$

(c)



20. $f_{\text{ave}} = \frac{1}{4-1}\int_1^4 f(x)\,dx = \frac{1}{3}\int_1^2 x^2\,dx + \frac{1}{3}\int_2^4 2x\,dx = \frac{1}{3}\left[\frac{1}{3}x^3\right]_1^2 + \frac{1}{3}\left[x^2\right]_2^4 = \frac{1}{9}(8-1) + \frac{1}{3}(16-4) = \frac{7}{9}+4 = \frac{43}{9}$

21. Use geometric interpretations to find the values of the integrals.

$\int_0^8 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx + \int_4^6 f(x)\,dx + \int_6^7 f(x)\,dx + \int_7^8 f(x)\,dx$

$= -\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1 + 4 + \frac{3}{2} + 2 = 9$

Thus, the average value of f on $[0,8] = f_{\text{ave}} = \frac{1}{8-0}\int_0^8 f(x)\,dx = \frac{1}{8}(9) = \frac{9}{8}$

23. $f_{\text{ave}} = \frac{1}{2-(-1)}\int_{-1}^{2}\left[3x^2+2x\right]dx = \frac{1}{3}\left[x^3+x^2\right]_{-1}^{2} = \frac{1}{3}(8+4) - \frac{1}{3}(-1+1) = 4,$ option (**A**).

24. The average value of a function over the interval $[-1,1]$ is $f_{\text{ave}} = \frac{1}{1-(-1)}\int_{-1}^{1} f(x)\,dx = \frac{1}{2}\int_{-1}^{1} f(x)\,dx.$

For $f(x) = x^3,\ f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} x^3\,dx = \frac{1}{2}\cdot\frac{1}{2}x^4\Big]_{-1}^{1} = \frac{1}{8}((1-1) = 0.$

For $f(x) = \sin x,\ f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} \sin x\,dx = -\frac{1}{2}\cos x\Big]_{-1}^{1} = -\frac{1}{2}(\cos 1 - \cos(-1)) = 0.$

For $f(x) = xe^{x^2},\ f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} xe^{x^2}\,dx$ $\left[\begin{array}{c} u = x^2, \\ \frac{1}{2}du = x\,dx \end{array}\right] = \frac{1}{2}\cdot\frac{1}{2}e^{x^2}\Big]_{-1}^{1} = \frac{1}{4}\left(e^{1-e^1}\right) = 0.$

But for (**B**), $f(x) = 3x^2,\ f_{\text{ave}} = \frac{1}{2}\int_{-1}^{1} 3x^2\,dx = \frac{1}{2}x^3\Big]_{-1}^{1} = \frac{1}{2}(1-(-1)) = 1 \neq 0.$

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9. $y = \sin x \Rightarrow dy/dx = \cos x \Rightarrow 1 + (dy/dx)^2 = 1 + \cos^2 x$. So $L = \int_0^{\pi} \sqrt{1+\cos^2 x}\, dx \approx 3.8202$.

11. $y = x - \ln x \Rightarrow dy/dx = 1 - 1/x \Rightarrow 1 + (dy/dx)^2 = 1 + (1-1/x)^2$. So $L = \int_1^4 \sqrt{1+(1-1/x)^2}\, dx \approx 3.4467$.

13. $x = \sqrt{y} - y \Rightarrow dx/dy = 1/(2\sqrt{y}) - 1 \Rightarrow 1 + (dx/dy)^2 = 1 + \left(\frac{1}{2\sqrt{y}} - 1\right)^2$.

So $L = \int_1^4 \sqrt{1+\left(\frac{1}{2\sqrt{y}}-1\right)^2}\, dy \approx 3.6095$.

15. $y = 1 + 6x^{3/2} \Rightarrow dy/dx = 9x^{1/2} \Rightarrow 1 + (dy/dx)^2 = 1 + 81x$.

So $L = \int_0^1 \sqrt{1+81x}\, dx = \int_1^{82} u^{1/2} \cdot \frac{1}{81} du = \frac{1}{81} \cdot \frac{2}{3} u^{3/2} \Big]_1^{82} = \frac{2}{243}\left(82\sqrt{82} - 1\right)$.

21. The line at the top of the region has length $3 - (-3) = 6$. Then for

$y = x^2 - 5,\ y' = 2x \Rightarrow 1 + (y')^2 = 1 + (2x)^2 = 1 + 4x^2$.

So the length of the curve is $L = \int_{-2}^{3} \sqrt{1+(2x)^2}\, dx = 2\int_0^3 \sqrt{1+4x^2}\, dx \overset{\text{CAS}}{\approx} 2(9.747088759) \approx 19.494$.

Thus the perimeter of the given region is $P \approx 25.494$.

30. If $f(x) = x^2 + 5x$, then $s(k) = \int_1^k \sqrt{1+[f'(x)]^2}\, dx = \int_1^k \sqrt{1+[2x+5]^2}\, dx = \int_1^k \sqrt{4x^2+20x+26}\, dx$.

Then $s'(k) = \frac{d}{dt}\left[\int_1^k \sqrt{4x^2+20x+26}\, dx\right] = \sqrt{4k^2+20k+26} \cdot \frac{dk}{dt} = 3\sqrt{4k^2+20k+26}$. When $k = 5$, the length of the arc is increasing at a rate of $3\sqrt{4 \cdot 25 + 100 + 26} = 3\sqrt{226}$, option (**A**).

31. $G(x) = \int_0^x \sqrt{t^2+6t+8}\, dt \Rightarrow G'(x) = \frac{d}{dx}\left[\int_0^x \sqrt{t^2+6t+8}\, dt\right] = \sqrt{x^2+6x+8}$ and

$1 + [G'(x)]^2 = 1 + x^2 + 6x + 8 = x^2 + 6x + 9 = (x+3)^2$. So the arc length for $2 \le x \le 4$ is

$L = \int_2^4 \sqrt{(x+3)^2}\, dx = \int_2^4 |x+3|\, dx = \int_2^4 (x+3)\, dx = \left[\frac{1}{2}x^2 + 3x\right]_2^4 = (8+12) - (2+6) = 12$, (**C**).