

Substitution

p. 467: 5-53 EOO

5. Let $u = 2x$. Then $du = 2dx$ and $dx = \frac{1}{2}du$, so $\int \cos 2x dx = \int \cos u \left(\frac{1}{2}du\right) = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2x + C$.

9. Let $u = x^4 - 5$. Then $du = 4x^3 dx$ and $x^3 dx = \frac{1}{4}du$,

$$\text{so } \int \frac{x^3}{x^4 - 5} dx = \int \frac{1}{u} \left(\frac{1}{4}du\right) = \frac{1}{4} \ln|u| + C = \frac{1}{4} \ln|x^4 - 5| + C.$$

13. Let $u = 1 - 2x$. Then $du = -2dx$ and

$$dx = -\frac{1}{2}du, \text{ so } \int (1 - 2x)^9 dx = \int u^9 \left(-\frac{1}{2}du\right) = -\frac{1}{2} \cdot \frac{1}{10} u^{10} + C = -\frac{1}{20} (1 - 2x)^{10} + C.$$

17. Let $u = 5 - 3x$. Then $du = -3dx$ and $dx = -\frac{1}{3}du$, so $\int \frac{dx}{5 - 3x} = \int \frac{du}{-3u} = -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|5 - 3x| + C$.

21. Let $x = 1 - e^u$. Then $dx = -e^u du$ and $e^u du = -dx$,

$$\text{so } \int \frac{e^u}{(1 - e^u)^2} du = \int -\frac{1}{x^2} dx = -\int x^{-2} dx = -(-x^{-1}) + C = \frac{1}{x} + C = \frac{1}{1 - e^u} + C.$$

25. Let $u = \ln x$. Then $du = \frac{dx}{x}$ so $\int \frac{(\ln x)^2}{x} dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$.

29. Let $u = 1 + e^x$. Then $u^2 = 1 + e^x$, and $2u du = e^x dx$, so

$$\int e^x \sqrt{1 + e^x} dx = \int u \cdot 2u du = \frac{2}{3}u^3 + C = \frac{2}{3}(1 + e^x)^{3/2} + C.$$

33. Let $u = 5^t$. Then $du = 5^t \ln 5 dt$ and $5^t dt = \frac{1}{\ln 5} du$,

$$\text{so } \int 5^t \sin(5^t) dt = \int \sin u \left(\frac{1}{\ln 5} du\right) = -\frac{1}{\ln 5} \cos u + C = -\frac{1}{\ln 5} \cos(5^t) + C.$$

37. Let $u = 1 + 5t$. Then $du = 5 dt$ and $dt = \frac{1}{5} du$,

$$\text{so } \int \cos(1 + 5t) dt = \int \frac{1}{5} \cos u du = \frac{1}{5} \sin u + C = \frac{1}{5} \sin(1 + 5t) + C.$$

41. Let $u = \sin x$. Then $du = \cos x dx$ so $\int \sqrt{\sin x} \cos x dx = \int \sqrt{u} du = \int u^{1/2} du = \frac{2}{3}u^{3/2} + C = \frac{2}{3} \sqrt{\sin^3 x} + C$.

45. $\int \cot x dx = \int \frac{\cos x}{\sin x} dx$. Let $u = \sin x$. Then $du = \cos x dx$,

$$\text{so } \int \cot x dx = \int \frac{1}{u} du = \ln|u| + C = \ln|\sin x| + C.$$

49. Let $u = 1 + x^2$. Then $du = 2x dx$,

$$\begin{aligned} \text{so } \int \frac{1+x}{1+x^2} dx &= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx = \tan^{-1} x + \int \frac{\frac{1}{2} du}{u} = \tan^{-1} x + \frac{1}{2} \ln|u| + C \\ &= \tan^{-1} x + \frac{1}{2} \ln|1+x^2| + C = \tan^{-1} x + \frac{1}{2} \ln(1+x^2) + C \quad [\text{since } 1+x^2 > 0]. \end{aligned}$$

53. Let $u = 2x^2 + 1$. Then $du = 4x dx$ so $\int x \sqrt{2x^2 + 1} dx = \int \sqrt{2x^2 + 1} x dx = \int \frac{1}{4} \sqrt{u} du$, which is option (C).